

# **An Approach to Private Equity Modeling: Managing the Uncertainty**

Private equity is an important part of institutional portfolios. It provides attractive opportunities for long-term investors to harvest the illiquidity premium over time and extract the value created by hands-on private equity managers. We offer insights into the optimal way of building an allocation to private equity.

Many long-term investors have been increasing their allocations to private equity strategies in an effort to augment their long-term returns and introduce a differentiated return stream into their portfolios. We believe private equity is an important part of institutional portfolios. It provides attractive opportunities for long-term investors to harvest the illiquidity premium over time and to extract the value created by hands-on private equity managers.

Private equity serves as a core part of equity portfolios—the primary driver of returns for long-term oriented portfolios. Adding private equity enhances the portfolio’s return capabilities, but also the associated risks. The 2008–2009 financial crisis highlighted the critical need for investors to understand the liquidity risk of private equity investments and to develop a disciplined asset allocation framework.

In this paper, we consider how to build and maintain a private equity portfolio. We present a model that offers insights into the asset allocation process. The model identifies a prudent upper bound for a private equity allocation—a point at which unfunded liabilities and illiquidity do not present significant risks to the portfolio. We conclude that a 50% allocation is feasible with an estimated unfunded ratio of about 23%. We also explore a strategy for pacing commitments that allocates to new private equity investments at a consistent annual rate. This approach allows the endowment to achieve its target allocation over time, while steadily building the portfolio across vintage years. Lastly, we consider how to invest committed capital before it is invested directly in private equity strategies, and conclude that investing in higher return/risk assets such as public equities is the most effective approach.

## Introduction

Many institutions focus solely on determining their private equity allocation targets without spending sufficient time developing a robust strategy for ensuring the targets are realized and maintained. Obtaining the benefits of an allocation to private equity, while also avoiding its inherent illiquidity pitfalls, can only occur through an effective, risk-based strategy for executing the build-out to the long-term equilibrium state.

The goal of our analysis is to develop a framework for determining the proper long-term private equity allocation target and to craft a sound approach for getting there. We focus on three key questions:

1. What is a **prudent upper bound** for the private equity allocation before liability and liquidity risks become a real threat?
2. What is the optimal way to reach the sustainable desired private equity allocation and get vintage year diversification? Specifically, what is the **annual commitment rate that leads to the target private equity allocation** over the long run?
3. In the process of ramping up a target private equity allocation, **where should the outstanding capital (the difference between the actual and the target private equity allocation) be temporarily invested?** Should it be invested in low-risk, low-return assets such as bonds or high-risk, high-return assets such as public equities?

The uncertainties around the rate and timing of the private equity managers' capital calls and distributions present challenges to this analysis. We use a Monte Carlo model with scenario-based outputs to better address these uncertainties and to help answer the three questions posed above.

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## The Model

To simplify the analysis, we present a generic portfolio model that holds only three asset classes: public equity, private equity, and fixed income. We assume returns follow a multivariate normal distribution and assume nominal expected returns of 7% and 3% for public equity and fixed income, with 15% and 5% volatilities, respectively. For the private equity portfolio, we assume a 5% annual illiquidity premium. We assume a spending rate of 5% based on the previous year's total portfolio value with the implicit assumption that the public equity and bond holdings provide sufficient liquidity for all annual cash flow and rebalancing needs. We begin with a zero percent realized private equity allocation and then build it up over time. Our analyses are drawn from 20,000 iterations on a 30-year horizon simulation to control statistical variability over the long run.

Estimating investment and holding periods for private equity investments is always challenging as available data sets are limited and these time periods are impacted by market conditions. To better anchor our work with real world experience, we rely on large sets of industry data across multiple market cycles and focus on mature US buyout and venture capital funds due to their dominant role in a typical private equity portfolio (**Appendix 1**). We assume a relatively large standard deviation of 10% for both capital calls and distribution rates to accommodate a wide range of experience with respect to the speeds at which capital is called, invested, and returned over time. We also consider correlations between capital calls, distributions, and public equity performance to develop the stress scenarios (**Appendix 2**).

In practice, because endowment investment officers have no control over the rate or the timing of private equity managers' capital calls and distributions, we focus on two key variables that they can control: the annual commitment rate and the risk profile of the assets waiting to be invested in private equity assets.

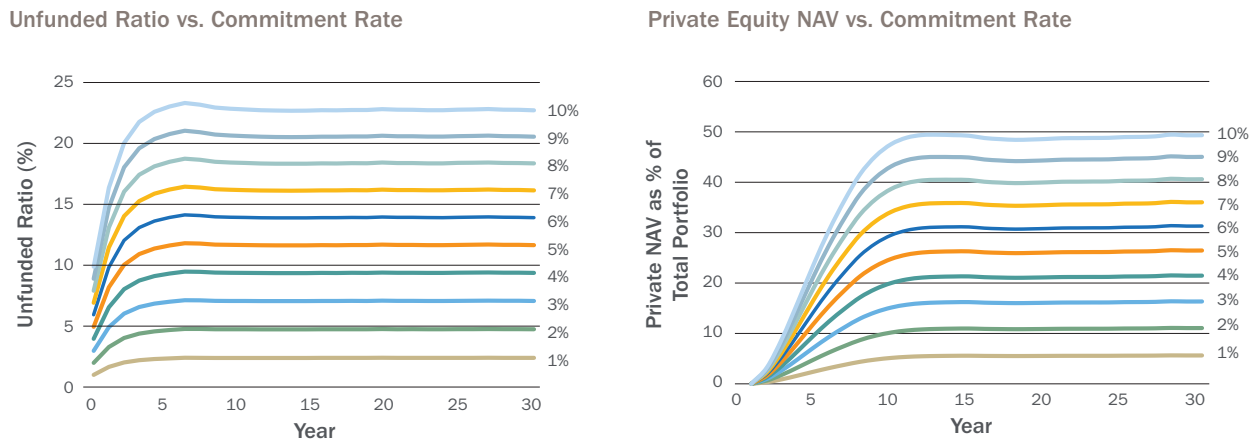
The annual commitment rate is the new commitment to private equity every year as a percentage of last year's total portfolio assets. Once selected, we keep the commitment rate constant each year in order to replicate the behavior of an endowment manager running an actual portfolio. An annual commitment rate results in a long-term equilibrium percentage of the portfolio in private equity assets, as well as the portfolio's corresponding unfunded commitment level. The unfunded commitment level is important from a risk perspective as it represents a nominal liability to fund future capital calls, regardless of the prevailing market environment at the time of capital calls. The desired risk profile of the capital awaiting investment in private equity (the difference between the modeled actual private equity allocation and the target private equity allocation) determines how much of that capital is invested in equities versus bonds. A risk profile of 100% equity means we put all of the capital into public equities (risk assets), while 0% means we put the entire amount into bonds (safe assets).

## The Annual Commitment Strategy

The first variable is the annual commitment rate into private equity strategies. Maintaining a consistent rate of commitment is critical to structuring a portfolio that is diversified across vintage years and takes advantage of varying market cycles. It is also important in terms of building up and maintaining a private equity portfolio in a prudent, gradual, and dynamic way.

**Figure 1** shows the base cases of the unfunded ratio and the private equity allocation reaching long-term equilibrium levels under different annual commitment rates over time (ranging from 1% to 10%). As a rule of thumb, at low rates, the equilibrium private equity allocation is about twice the unfunded ratio, e.g., a 6% annual commitment rate leads to a base case unfunded ratio around 15% (**left chart**) and a private equity allocation of around 30% at equilibrium (**right chart**).<sup>1</sup>

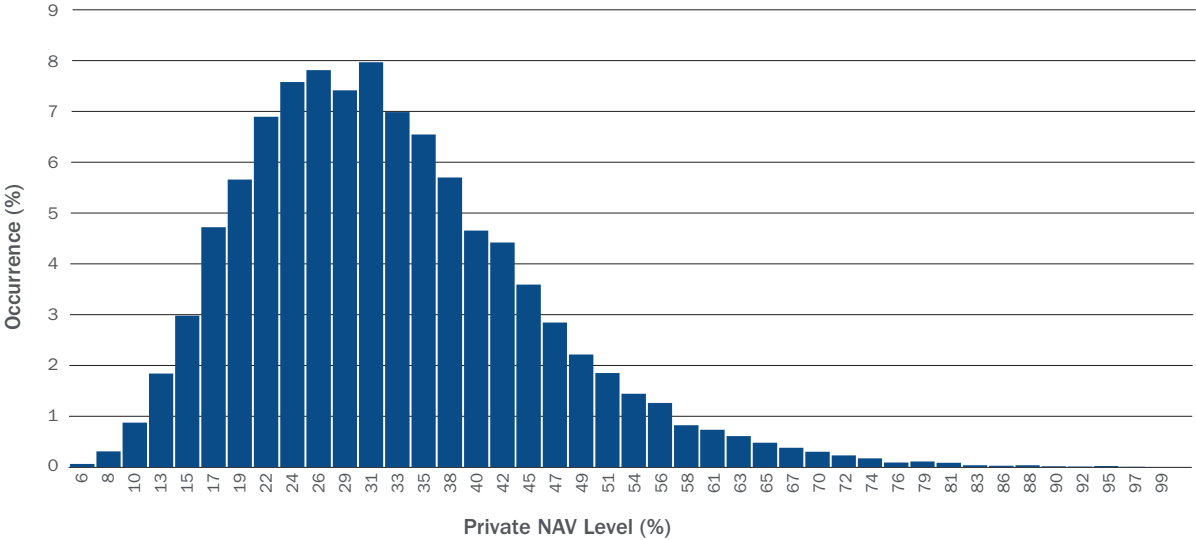
**Figure 1: Building Up a Private Equity Portfolio**



**Figure 1** serves as a useful guide for investment officers who are trying to build up their private equity portfolios smoothly over time by committing at the same rate each year.<sup>2</sup> For example, if the investment officer is targeting a 30% private allocation over the long run, the model suggests making 6% in new commitments every year. The curves in **Figure 1** represent the base case values, which are the averages of 20,000 scenarios. In other words, there are 20,000 outcomes based on different paths that the markets and private equity managers could take.

Figure 2 shows all of the possible private equity allocations at year 30 under the 6% annual commitment rate.<sup>3</sup> We can see that there is about a 24% chance of a private equity allocation bigger than 40% at year 30, a scenario that corresponds to distressed public markets and/or slow distributions from the private equity managers.

Figure 2: Distribution of Private Equity NAV Levels at Year 30 Assuming a 6% Annual Commitment Rate

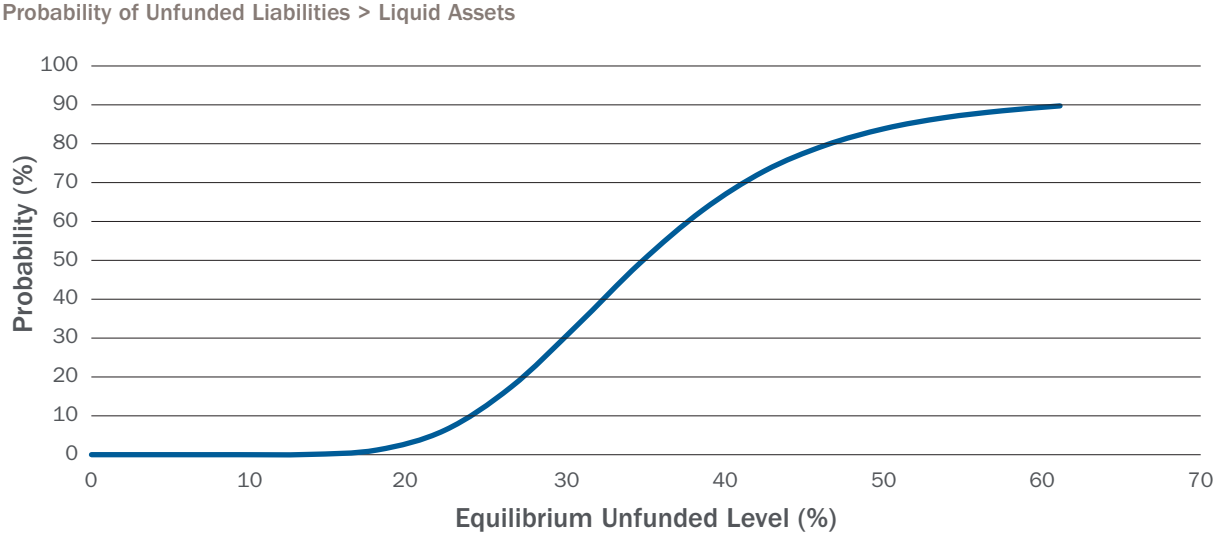


The distribution-based outcomes are informative as they allow us to think about and define risks in terms of probabilities, for example, the probability of unfunded liabilities being greater than liquid assets (the sum of public equities and bonds).

There is about a 24% chance of a private equity allocation bigger than 40% at year 30, a scenario that corresponds to distressed public markets and/or slow distributions from the private equity managers.

**Figure 3** shows this risk at equilibrium versus various unfunded levels.<sup>4</sup> The model suggests that the liability-related risk defined here is quite low when the equilibrium unfunded level is below 23%, which translates into an annual commitment rate of 10% and a target private equity allocation of about 50%.

**Figure 3: Liability Risk**



Thus far we have established that it is quantitatively feasible to have a target private equity allocation below 50% and an unfunded ratio below 23%, taking into account liability-related risks. In reality, given the limited opportunity size, liquidity constraints, and possible rebalancing risks, we think it is generally more prudent to have a target private equity allocation in the range of 30% to 40%. Also, we believe the best way to reach the target allocation is through steady annual commitments following the rates suggested by **Figure 1**.

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## The Risk Profile of the Capital Awaiting Investment in Private Equity

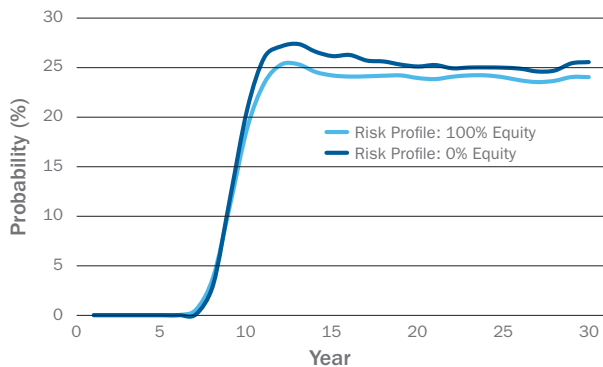
We now focus on the case of a 6% commitment rate (which **Figure 1** shows corresponds to a 30% target private equity allocation) to study the impact of the investment officer’s second control variable: the risk profile of the capital that is waiting to be invested in the target private equity allocation. We also introduce another measure of risk related to rebalancing and liquidity: the probability of an actual private equity allocation being larger than a certain threshold.

Private equity is inherently illiquid and difficult to rebalance, as evidenced by the high costs associated with sales in the secondary markets (difficult to value, likely will need to be sold at a discount, long lead times before sales, search costs, etc.), especially under stressed market conditions. A private equity allocation that is too large poses a danger to the total portfolio, not only by altering its liquidity profile, but also by affecting its equity/bond ratio, as it is difficult to rebalance or reduce the private equity exposure. As an extreme example, a private equity allocation of 70% implies a 0% allocation to public equities if one wants to keep the overall equity/bond balance at a 70%/30% ratio. Furthermore, any increase in the value of the private equity could potentially surpass the total equity target of the portfolio.

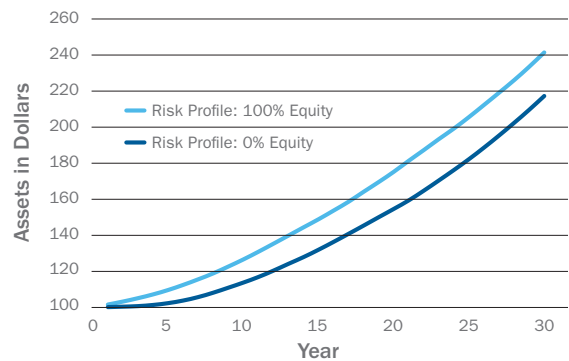
Here we use a threshold of 40% to show the nature of the liquidity risk profile over time. **Figure 4 (left)** shows that the liquidity risks (defined as a private equity allocation greater than 40%) of the two portfolios are quite similar over time.<sup>5</sup>

**Figure 4: Risk and Return Trade-Offs Under Different Risk Profiles**

Probability of Private Equity NAV > 40%  
At a 6% Annual Commitment Rate



Total Portfolio Assets  
At a 6% Annual Commitment Rate





In the early years, the portfolio with a risk profile of 100% equity (100% of the difference between the actual and target private equity allocation is invested in public equity) has slightly higher liquidity risk, due to the higher volatility of public equities as compared with bonds. However, because the private equity allocation in the early years is so low, the probability of an actual private equity allocation larger than 40% (liquidity risk) is virtually zero for both portfolios. In the later years, as the actual private equity allocation grows and becomes more significant, the liquidity risk increases dramatically for both portfolios. Again, however, the difference is trivial. Although a bond allocation offers lower volatility, it also has a much larger return drag. In other words, the total liquid part of the portfolio (public equities plus bonds) with a risk profile of 0% equity grows at a much slower rate than with a risk profile of 100% equity, and lags the private equity allocation by much more.

The trade-off between return and volatility results in a similar overall portfolio liquidity risk profile. It is worth noting that in practice, we model and monitor several thresholds above the target allocation. For example, under a 6% annual commitment rate, the probabilities of an actual private equity allocation larger than 40%, 45% and 70% at year 30 are about 24%, 15% and 1%, respectively (recall **Figure 2**). We think these probabilities are low and manageable.

From **Figure 4 (right)**, we can see that the higher the risk profile of the capital awaiting investment in private equity strategies, the higher the total portfolio returns, especially during the early period when the actual private equity allocation is low.<sup>6</sup> A risk profile of 100% equity for the capital awaiting investment is equivalent to a 70% equity/30% bond mix for the overall portfolio at year 0 (assuming a 0% actual private equity allocation), while a risk profile of 0% equity is equivalent to a 40% equity/60% bond mix for the overall portfolio.<sup>7</sup> Such low allocations to equity at low-risk profiles might not be consistent with the risk tolerance and return objective of long-term investors such as endowments.<sup>8</sup>

The return difference in the early days would cause a significant return drag to the 0% risk profile portfolio over the long run. At year 30, the ending asset value of the high-risk (100% equity) portfolio is more than 11% higher than that of the low-risk profile (0% equity) portfolio. This return shortfall is likely unacceptable to institutions seeking the return generation and intergenerational equity that a meaningful long-term allocation to private equity is expected to provide.

## Conclusion

In this paper, we have presented an analysis that is important to managers of both young and mature private equity programs. We propose a Monte Carlo simulation-based model as a tool for providing guidance on both building up and managing a private equity allocation for a typical endowment. One of the many unique characteristics of private equity is the fact that allocated commitments do not necessarily translate to an equal amount of capital at work, which makes forecasting future exposures difficult. Prevailing market conditions, fundraising cycles, sub-strategy characteristics, and manager-specific issues all influence the pace at which capital is drawn, which introduces ambiguity into portfolio planning efforts. Our scenario-based outputs address the uncertainties around the timing and rate of capital calls and distributions.

The model suggests that a target allocation to private equity strategies in the range of 30% to 40% presents minimal liability and liquidity risks. Using an even annual commitment rate is a reasonable way to achieve a target allocation, while also reducing vintage year risks. Lastly, for private equity programs that are being ramped up (i.e., those that have not yet reached their target allocations), our research suggests that it is optimal for long-term investors such as endowments to invest the capital awaiting investment in private equity strategies in risk assets with higher expected returns, such as public equities.

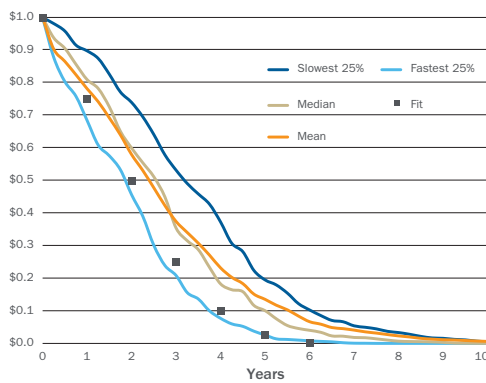
Many of these insights require expertise and resources to implement in an effective, customized way. While we believe that long-term investors such as endowments are uniquely positioned to capture a premium from the illiquidity of private equity investments, it is important to do so in a risk-controlled way, starting with asset allocation modeling and continuing throughout the commitment period.

**The model suggests that a target allocation to private equity strategies in the range of 30% to 40% presents minimal liability and liquidity risks.**

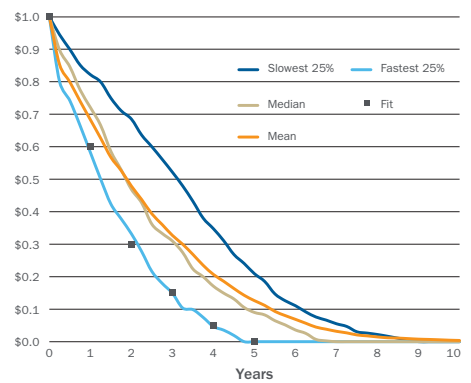
## Appendix 1

We focus on mature US buyout and venture capital funds with complete data over the funds' lives. Our pool contains 119 buyout and 188 venture capital funds. All the cash flow and NAV data are normalized to a \$1 total commitment for each fund. We look at how key measurements, including unfunded commitment, NAV, cumulative distribution and total return (cumulative distribution + NAV) change over time.

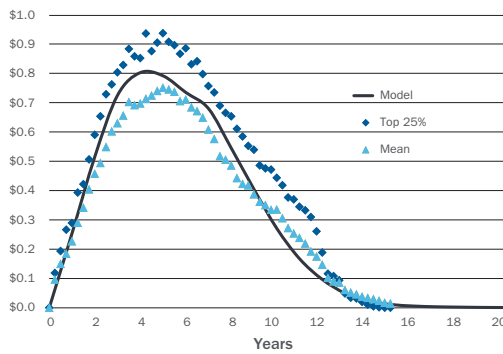
Buyout Unfunded



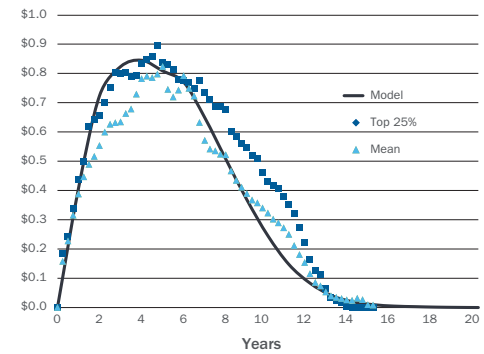
Venture Capital Unfunded



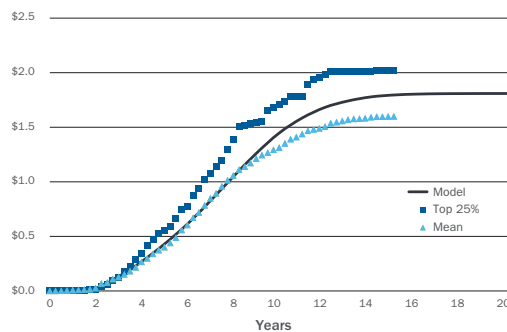
Buyout NAV



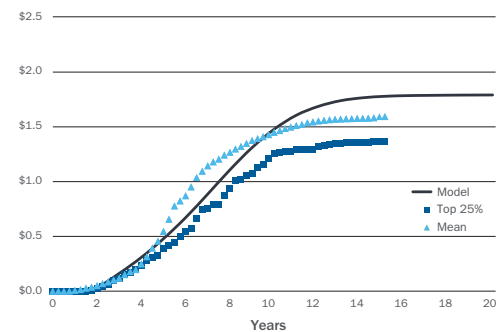
Venture Capital NAV



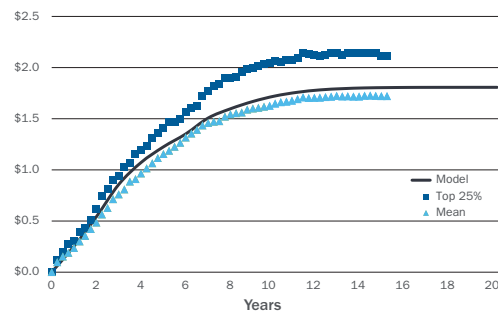
Buyout Cumulative Distribution



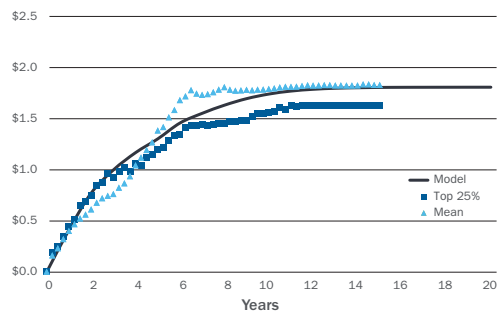
Venture Capital Cumulative Distribution



Buyout Total Return



Venture Capital Total Return



The modeled rates of capital calls are in-line with the fastest 25% of funds measured by their draw down of total commitments, based on both historical industry data as well as our own experience. For example, the buyout fund is modeled on the average to draw down 25% of the fund in each of the first three years and 90% by year 4. It leaves a 10% “tail” amount for the last two years. Compared to buyouts, venture capital has a faster initial capital call rate. The blended private equity scenario is the final input we used for the model, which represents a typical private equity portfolio that is a balanced mix of buyout and venture capital funds. The annual distribution rate of 11.11% ( $=1/9$ ) is fitted so the NAV and cumulative return curves lie between the mean and top 25% performers in the data set, given the capital call rates decided in the previous step. These parameters should be fine-tuned for different portfolios with different mixes of buyout and venture capital, managers that may invest faster or more slowly, etc. Our goal is to provide a practical framework to study the data and model the risks. Our example should be viewed as a general case to demonstrate the usefulness of such a framework.

Year	Buyout	Venture Capital	Blended PE
1	25.00%	40.00%	32.50%
2	25.00	30.00	27.50
3	25.00	15.00	20.00
4	15.00	10.00	12.50
5	7.50	5.00	6.25
6	2.50		1.25
<b>Total</b>	<b>100.00%</b>	<b>100.00%</b>	<b>100.00%</b>

## Appendix 2

We assume a -0.5 correlation between the capital call rate and public equity performance and a +0.2 correlation between the distribution rate and public equity performance, as an effort to model the extra stress from private equity funds calling capital faster and returning capital slower when the public markets are doing poorly. The results are not sensitive to these correlations. Other reasonable correlations yield consistent results. Also we allow for wide standard deviations in calls and distributions.

Correlation Assumptions				
	Equities	Bonds	Capital Call	Distribution
Global Equities	1.0	0.1	-0.5	0.2
U.S. Agg. Bonds		1.0	0.0	0.0
Capital Calls			1.0	0.0
Distributions				1.0

### Endnotes

- <sup>1</sup> Due to its nonlinear nature, this relationship will break down at extreme levels, such as unfunded ratios above 50%, because the maximum private equity market value is 100%. Also, it relies on assumptions of the investment and holding periods. For example, if the private equity portfolio invests only in venture capital, which typically has a relatively short investment period but a long holding period, the relationship will change. Results are available on request.
- <sup>2</sup> Timing (vintage year) risk is reduced by making even commitments every year.
- <sup>3</sup> Modeled with a risk profile of 50% equity.
- <sup>4</sup> To be exact, it is the average of the readings from year 20 to year 30. Again, we observe the nonlinear nature of the curves. We used a risk profile of 100% equity for this analysis, but analyses using other risk profiles, such as 0% and 50%, produce similar results.
- <sup>5</sup> As mentioned before, different risk profiles result in similar curves in Figure 3, regarding liability-related risks.
- <sup>6</sup> The total portfolio returns (slopes of the curves) differ the most in the beginning when the realized private equity allocation is low. Eventually, the slopes of the curves will converge as the target private equity allocation is achieved and the difference between the actual private equity allocation and the target private allocation is zero.
- <sup>7</sup> The difference in annual expected returns from the two portfolios is 1.2%, based on our assumptions. Meaning that the annual return for the private equity portfolio is always 5% higher than that of the public equity portfolio.
- <sup>8</sup> See “Asset Allocation with Short-Term and Long-Term Risk Objectives,” TIAA Endowment & Philanthropic Services, 2018.

Please note the Monte Carlo modeling presented above is subject to several limitations including: for simplicity, we use normal distributions for asset class returns and further assume that they are independent and identically distributed (other specific distributions with fat tails and serial correlations can be incorporated on request). Correlations between asset classes vary over time and can be changed and modeled dynamically. The 5% illiquidity premium is net of all fees.



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