

# Fixed and variable longevity annuities in defined contribution plans: Optimal retirement portfolios taking Social Security into account

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## Abstract

This paper investigates retirees' optimal purchases of fixed and variable longevity income annuities using their defined contribution (DC) plan assets and given their expected social security benefits. As an alternative, we also evaluate using plan assets to boost social security benefits through delayed claiming. We determine that including deferred income annuities in DC accounts is welfare enhancing for all sex/education groups examined. We also show that providing access to variable deferred annuities with some equity exposure (similar to participating annuities) further enhances retiree wellbeing, compared to having access only to fixed annuities. Nevertheless, for the least educated, delaying claiming social security benefits is preferred, whereas the most educated benefit more from using accumulated DC plan assets to purchase deferred annuities.

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## Introduction

The US social security retirement system pays retirees a lifetime annuity with fixed real benefits that depend (progressively) on retirees' earning histories and claiming ages. For this reason, if a retiree receives a substantial portion of her income through a social security annuity, it stands to reason that her remaining financial portfolio should include substantial exposure to risky equities, through a target date fund or with annuities whose payments are linked at least in part to the performance of an equity portfolio. Additionally, social security replacement rates are higher for lifetime low-earners, and lower for lifetime high-earners. As a result, low lifetime earners receiving a higher replacement rate could decide to devote a greater proportion of their remaining financial wealth to risky equities, through a target date fund or direct equity investment. Conversely, higher lifetime-earning retirees receiving a relatively low social security replacement rate may wish to purchase larger private annuities in old age from their tax-qualified retirement accounts, to provide a predictable income stream sufficient to cover necessities. Moreover, the 2019 SECURE Act encouraged the supply of annuities in defined contribution (DC) plans and Individual Retirement Plans (IRAs) worth \$21.3 trillion (ICI 2021), providing plan sponsors "safe harbor" rules for their inclusion.

This paper focuses on how these two instruments—annuities with lifelong benefits purchased using DC plan assets, and social security annuities—should be considered jointly to optimize household welfare. Understanding how these interact is of key importance in order to generate efficient retirement portfolios. Additionally, there is likely to be substantial heterogeneity in the demand for longevity annuities across the retiree population, depending on their assets inside and outside tax-qualified retirement plans, their mortality assumptions, and their accrued social security benefits.

Federal regulation requires compliance with various conditions for the purchase of an annuity to qualify for favorable tax treatment (known as a qualifying life annuity contract, or QLAC). The purchase price of the QLAC cannot exceed 25% of the assets in the DC plan and, at the same time, cannot exceed \$130,000; moreover, the lifetime income payments must begin no later than age 85. Except for a refundable premium option, the QLAC may not include death benefits. Of particular interest in this paper are the regulations regarding the nature of lifetime payments. The Internal Revenue Service explicitly stated (Federal Register Vol. 79, No. 12/, July 2, 2014)

that QLACs must generally make predictable (nominal or real) fixed lifetime payments. Linking payments to a stock market index or to a portfolio of mutual funds is expressly not permitted, even if there is a minimum guaranteed income under those contracts, also known as variable or investment-linked annuities. Interestingly, however, participating life annuities, where payments are linked to the overall investment experience of a life insurance company, are consistent with the regulatory requirements of a QLAC.

In particular, we analyze the following research questions: What should be the retiree's optimal portfolio fraction of DC plan assets allocated to longevity annuities, bonds, and risky stocks? When should DC plan assets be used to permit retirees to (a) defer claiming social security benefits and cover spending needs in the meanwhile, or (b) buy a deferred annuity while simultaneously claiming social security benefits? To what extent should a longevity annuity in a DC plan offer fixed versus variable payouts, where the latter are linked to the return of an underlying portfolio of risky stocks and bonds (e.g., in a participating annuity or investment-linked annuity)? What welfare gains are feasible, given these annuities? We also examine how the selected deferral period of the QLAC alters outcomes. Should it be short, as in the case of an immediate annuity; should the retiree select the maximum possible regulatory deferral period until age 85; or is the better deferral period fall between these two boundary cases?

There are three literatures to which our work is related: economic studies on life cycle financial decisionmaking, the decisions to purchase annuities in retirement, and delayed claiming of social security benefits. Excellent reviews of the first area include Gomes (2020) and Gomes et al. (2021) who discuss dynamic consumption and portfolio choice models in discrete time. For the second area, we build on previous studies about the optimal demand for annuities (e.g., Huang et al. 2017; Horneff et al. 2010, 2020; Inkmann et al. 2011; Milevsky 2005) by exploring different deferral ages for the lifelong annuity. A third literature discusses the pros and cons of delaying social security claiming (e.g., Hubener et al. 2010; Shoven and Slavov 2014). We bring these three threads together by integrating the decision to delay claiming and annuitization. Closest to our work is Munnell et al. (2022) who discusses the possibility of using DC plan assets at retirement to finance delayed claiming or buy fixed annuities. Compared to the latter paper, our contributions are to embed the decisions in a full life

cycle model, which starts at age 25 and runs until 100, which incorporates optimal saving and investing across bonds and risky stocks, consumption, and withdrawal patterns for assets inside as well as outside the DC plan. Moreover, we investigate optimal annuitization ratios for both fixed as well as variable annuities and alternative deferral ages. Our model also includes heterogeneity in lifetime earnings, assets, and mortality across education groups, and importantly, we incorporate the rich institutional details including the progressive and complex US income tax code.

In what follows, we outline the methodological foundations of our life cycle model which we use to answer these questions. Subsequently we illustrate how we realistically calibrate the model parameters, and we use a matching procedure to select preference parameters so that the model results match the empirically observable assets invested by US workers in tax-qualified defined contribution retirement plans as closely as possible. Next we use our model to analyze the demand for and welfare consequences of four alternative settings: claiming social security at age 66 or 67 without access to deferred income annuities (DIAs), and claiming social security at age 66 with access to fixed and variable DIAs. We extend prior research by comparing the value of purchasing private annuities, versus delaying social security benefits using assets from their DC accounts to finance consumption. We document that using retirement account assets to purchase at least some fixed deferred income annuities is welfare enhancing for all sex/education groups examined, and allowing payout annuities to have a small exposure to equity can further enhance welfare. Nevertheless, for the least educated, delaying claiming social security benefits is preferred, whereas the most educated benefit more from using accumulated DC plan assets to purchase deferred annuities.

## I. Life cycle model: Methodology

Our discrete time dynamic portfolio and consumption model posits a utility-maximizing worker who decides how much to consume optimally and how much to invest in risky stocks, bonds, and annuities over her lifetime. We model utility as depending on consumption and bequests, while constraints include a realistic characterization of income profiles, taxes, and the opportunity to invest in risky stocks and riskless bonds both in a DC tax-qualified retirement plan (up to a limit) as well as in non-tax-qualified accounts. At retirement

(assumed here to be age 66), the individual determines how much of her retirement account she wishes to convert to a deferred income annuity (DIA), with the remainder held in stocks and bonds. We also take into account the Required Minimum Distribution rules relevant to the US DC setting, as well as a realistic formulation of social security benefits. We use mortality heterogeneity across educational categories.<sup>1</sup>

### A. Preferences

The individual's decision period starts at  $t = 1$  (age of 25) and ends at  $T = 76$  (age 100); accordingly, each period corresponds to a year. The individual's subjective probability of survival from time  $t$  until  $t + 1$  is denoted by  $p_t^s$ . Preferences at time  $t$  are specified by a time-separable *Epstein-Zin* utility function defined over current consumption,  $C_t$ . The parameter  $\rho$  represents the coefficient of relative risk aversion,  $\psi$  the elasticity of intertemporal substitution (EIS) and  $\beta$  is the time preference rate on future utility. The term  $Q_{t+1}$  denotes the level of bequest at time  $t+1$ . The strength of Individual bequest motive is controlled by variable  $b$ . Then the recursive definition of the corresponding value function is given by:

$$J_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta \left( \sum_i \Pi_{ij,t} E_t \left( p_t^s J_{t+1}^{1-\rho} + (1 - p_t^s) b \left( \frac{Q_{t+1}}{b} \right)^{1-\rho} \right) \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \quad (1)$$

where the  $\Pi_{ij,t} = \text{Prob}(l_{t+1} = i | l_t = j)$  is a time-dependent transition matrix representing the probability to move from current ( $t$ ) income level  $j$  into income level  $i$  one year later ( $t + 1$ ).

1 Throughout this paper, we work in real terms (e.g., for labor income and asset returns). This is justified as the social security bend points, the brackets for income taxation, and the maximum contribution limits to retirement plans are basically adjusted for inflation annually.

## B. Annuity: Pricing and payouts

At age 66 ( $K$ ), the individual determines how much (up to 25%) of her DC plan assets ( $DIA_K \leq \min(0.25L_K, \$130,000)$ ) she will switch to a deferred annuity paying lifetime income benefits starting at age  $\tau$  (age 67, 80, or 85). The idea of using small amounts of accumulated assets to purchase deferred annuities was originally proposed by Milevsky (2005), who favored these over a more costly single premium immediate annuity. We consider two alternative products.

**Fixed annuity:** In case of a fixed longevity annuity purchased at age  $K$  for a nonrefundable premium of  $DIA_K$ , the life insurer starts paying fixed lifelong benefits (FPA) from age  $\tau$  as follows:

$$FPA_t = \frac{DIA_K}{\ddot{a}_\tau} \quad (t \geq \tau) \quad (2)$$

where  $\ddot{a}_{K,\tau} = p_{K,\tau}^a \sum_{s=0}^{121-\tau} (\prod_{i=\tau}^{\tau+s} p_i^a) R_f^{-(s+(\tau-K))}$  is the annuity factor. Here,  $R_f$  is the interest rate and  $p_i^a$  are the yearly survival probabilities the insurance company uses to price the annuity. These probabilities are derived from an actuarial mortality table and they may differ from the subjective survival probabilities used in the utility function. Moreover,  $p_{K,\tau}^a$  is the cumulative probability of surviving from age  $K$  until the end of the deferral period  $\tau$ .

**Variable annuity:** A variable payout annuity is a financial contract between a retiree and a life insurance company, whereby in exchange for paying an initial nonrefundable premium, the annuitant begins to receive lifelong payments when the deferral period is over, equal to the value of a pre-specified number of units in a specific asset portfolio (stocks, bonds, or some combination) represented by a mutual fund, index fund, or exchange-traded fund. Since payments depend on the value of the annuity fund units, they will be stochastic when the underlying portfolios hold risky assets. Hence, variable longevity annuities offer both an investment element, in terms of a mutual fund-style subaccount, and an insurance element, in terms of pooling longevity risks across the retiree group. Following Maurer et al. (2013), the payouts from the variable annuity (VPA) purchased at age  $K$  which start at age  $\tau$  are as follows:

$$VPA_{t+1} = \frac{VPA_t}{AIR} R_{t+1}^P \quad t \geq \tau \quad (3)$$

Here,  $R_{t+1}^P = R_f + \alpha_t(R_{t+1} - R_f)$  denotes the annual gross return of a portfolio invested in risky stocks with a share of  $\alpha_t$  and  $(1 - \alpha_t)$  in bonds. The first payment after the end of the deferral period is given

by  $VPA_\tau = \frac{DIA}{N_{K,\tau}} * \prod_{i=K}^{\tau} R_i^P$ . Here  $N_{K,\tau} = p_{K,\tau}^a \sum_{s=0}^{121-\tau} (\prod_{i=\tau}^{\tau+s} p_i^a) AIR^{-(s)}$  is the annuity factor, and where  $\prod_{i=K}^{\tau} R_i^P$  is the cumulative performance of the underlying asset portfolio within the deferral period. The payouts (after the deferral period) from the variable annuity are given by an updating rule that relates the annuity payouts  $VPA_{t+1}$  in the next year to the previous payout  $VPA_t$  and the realized investment return  $R_{t+1}^P$  on the underlying portfolio relative to the assumed interest rate ( $AIR$ ) set by the insurance company.<sup>2</sup> The annuity payment rises when  $R_{t+1}^P > AIR$ , it falls when  $R_{t+1}^P < AIR$ , and it is constant when  $R_{t+1}^P = AIR$ .

## C. Cash on hand

While working, the individual has the opportunity to invest a portion ( $A_t$ ) of her uncertain pre-tax salary  $Y_t$  (up to an annual limit of \$18,000)<sup>3</sup> in a tax-qualified retirement plan as well as in non-tax-qualified risky stocks  $S_t$  and risk free bonds  $B_t$ :

$$X_t = \begin{cases} C_t + S_t + B_t + A_t, & t < K \\ C_t + S_t + B_t, & t \geq K \end{cases} \quad (4)$$

Here  $X_t$  is after-tax cash on hand,  $C_t$  denotes consumption, and  $C_t, A_t, S_t, B_t \geq 0$  and  $A_t \leq \$18,000$  to age 51; additional retirement plan ‘catch-up’ contributions are permitted over age 51 of up to \$6,000. One year later, her cash on hand is given by the value of her stocks having earned an uncertain (real) gross return  $R_t$ , bonds having earned a riskless return of  $R_f$ , labor income  $Y_{t+1}$  reduced by housing costs  $h_t$  modeled as a percentage of labor income (as in Love 2010), and withdrawals ( $W_t$ ) from her DC plan, where withdrawals before age 59 1/2 result in a 10% penalty tax and are restricted to certain amount.<sup>4</sup>

2 See Horneff et al. (2010) for a detailed discussion of the role of the AIR in the annuity context.

3 The \$18,000 limit was the legal limit on U.S. tax-deferred contributions to DC plans in 2017; also, if the worker's plan permitted by the plan, employees age 50+ could make additional catch-up contributions of \$6,000 per year; see IRS (2017).

4 In the case of matching contributions, early withdrawals from tax-qualified retirement plans before age 60 usually require the employer's approval, typically given only in exceptional circumstances (hardship) and within limits. To address this, our model permits early withdrawals prior to age 60 if the worker has less than \$20,000 cash on hand; early withdrawals of up to half of the retirement plan assets are allowed. Both a 10% penalty tax and income tax must be paid on such withdrawals.

$$X_{t+1} = \begin{cases} S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} - Y_{t+1} d_w, & t < K \\ S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} - Y_{t+1} d_r, & K \leq t < \tau \\ S_t R_{t+1} + B_t R_f + Y_{t+1}(1 - h_t) + W_t - IT_{t+1}^{tax} + PA_t - Y_{t+1} d_r, & t \geq \tau \end{cases} \quad (5)$$

During her worklife, the individual pays taxes which reduce her cash on hand available for consumption and investment. We posit that labor income is reduced by 11.65% ( $d_w$ ), which is the sum of the Medicare (1.45%), city/state (4%), and Social Security (6.2%) taxes (up to a cap of 127,200 per year). In addition, the worker also must pay income taxes ( $IT_{t+1}^{tax}$ ) according to US federal progressive tax system rules (for details, see IRS 2017 and Appendix A).

The individual may save in a tax-qualified DC plan only during the working period, while non-pension saving in bonds and stocks is allowed over the entire life cycle. We model the exogenously-determined labor income process as  $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$  with a deterministic trend  $f(t)$ , permanent income component  $P_{t+1} = P_t \cdot N_{t+1}$ , and transitory shock  $U_{t+1}$ , using data from the Panel Study of Income Dynamics (PSID).<sup>5</sup> In our life cycle model, we work with a discrete Markov-switching income process,  $Y_{t+1} = I_{t+1}^l \cdot U_{t+1}^{MS}$ , for three income profiles each, for men and women. The transitory shocks  $U_{t+1}^{MS}$  depend on age, sex, and income by educational categories.

During retirement, the individual saves in stocks and bonds, and she also consumes. The DIA pays lifelong benefits from age  $\tau$  onwards. In retirement, the individual receives lifelong social security benefits determined by her Primary Insurance Amount (PIA) which is a function of her average indexed lifetime earnings (the AIME).<sup>6</sup> Her social security payments ( $Y_{t+1}$ ) in retirement ( $t \geq K$ ) are given by:

$$Y_{t+1} = PIA_K^l \cdot \varepsilon_{t+1}, \quad (6)$$

where  $\varepsilon_t$  is a lognormally-distributed transitory shock  $\ln(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$  with a mean of one which reflects out-of-pocket medical and other expenditure shocks (Love 2010).<sup>7</sup> During retirement, Social Security benefits are taxed (up to a limit)<sup>8</sup> at the individual federal income tax rate as well as the city/state/Medicare tax rate.

#### D. The tax qualified DC plan

Prior to retirement, the retiree's total value ( $L_{t+1}$ ) of her DC assets at time  $t + 1$  (for  $t < K$ ) is, therefore, determined by her previous period's value, minus any withdrawals ( $W_t \leq L_t$ ), plus additional contributions ( $A_t$ ) and returns from stocks and bonds:

$$L_{t+1} = \begin{cases} \omega_t^S (L_t - W_t + A_t + M_t) R_{t+1} + (1 - \omega_t^S) (L_t - W_t + A_t + M_t) R_f, & t < K \\ \omega_K^S (L_K - W_K - DIA_K) R_{K+1} + (1 - \omega_K^S) (L_K - W_K - DIA_K) R_f, & t = K \\ \omega_t^S (L_t - W_t) R_{t+1} + (1 - \omega_t^S) (L_t - W_t) R_f, & t \geq K \end{cases} \quad (7)$$

5 The PSID is a project of the National Institute on Aging, fielded at the University of Michigan; for more information see Panel Study of Income Dynamics (PSID) | National Institute on Aging (nih.gov).

6 The social security benefit formula is a piece-wise linear function of the Average Indexed Monthly Earnings providing a replacement rate of 90% up to a first bend point, 32% between the first and a second bend point, and 15% above that.

7 The assumed transitory variances are  $\sigma_\varepsilon^2 = 0.0784$  for high school and less than high school graduates, and  $\sigma_\varepsilon^2 = 0.0767$  for college graduates (Love 2010).

8 For details on how we treat social security benefit taxation, see the Appendix A. Due to quite generous allowances, relatively few individuals pay income taxes on their social security benefits.



The retirement plan assets are invested in a Target Date Fund having a relative stock exposure ( $\omega_t^s$ ) that declines according to age, following the popular “125–Age rule” ( $\omega_t^s = (125 - \text{Age})/100$ ).<sup>9</sup>

To be considered as a safe harbor DC plan and, therefore, avoid complex non-discrimination testing, we assume that the employers match 100% of employee contributions up to 5% of yearly labor income.<sup>10</sup> Due to regulation, the matching rate was applied only to a maximum compensation of \$270,000 (in 2017), so the maximum employer contribution was \$13,500. The matching contribution is then given by:

$$M_t = \min(A_t, 0.05Y_t, \$13,500). \quad (8)$$

The amount used to buy the DIA reduces the value of her DC assets invested in stocks and bonds. Wealth dynamics of the DC account are given by the previous value  $L_t$ , withdrawals  $W_t$ , and investment returns on stocks and bonds.

The US Treasury stipulates that DC participants take required minimum withdrawals (RMDs) from their plans from age 70.5 onwards or else they must pay a substantial tax penalty (50%); these are defined as a specified age-dependent percentage ( $m_t$ ) of plan assets.<sup>11</sup> Yet the value of the DIA is excluded when determining the retiree’s RMD. Therefore, to avoid the excise penalty, plan payouts are set so that  $mL_t \leq W_t < L_t$ . Benefit payments from the deferred annuity are counted in taxable income. It should be noted that at present, US regulation allows only fixed annuities to be excluded from the RMD calculation. Nevertheless, in our analysis below, we also examine outcomes if variable annuities were eligible for QLAC treatment, for purposes of comparison as a policy experiment.

## II. Model calibration

To calibrate the model, we use financial market parameters of the risk-free interest rate at 1% and an equity risk premium of 4% with a return volatility of 18%. For the variable annuity, the assumed interest rate is set equal  $AIR$  is 2%, and we consider three allocations to equities 50%, 20%, and according to the 125-age rule. Survival rates taken from the US Population Life Table 2017 (Arias and Xu 2019) with heterogeneity across sex/educational categories (Krueger et al. 2015), and for annuity pricing, we use the US Annuity 2012 mortality table (unisex, trend function until 2017) provided by the Society of Actuaries (SOA 2012). Annuity survival rates are higher than those for the general population because

they account for adverse selection among annuity purchasers. Social security old age benefits are based on the 35 best years of income and the bend points as of 2017 (US SSA nd\_a, nd\_b).<sup>12</sup> The age-dependent percentages ( $m_t$ ) of Required Minimum Distributions from DC plans are calculated as one divided by the retirees remaining life expectancy using the Internal Revenue Service (IRS 2015). In line with US rules, federal income taxes are calculated based on the household’s taxable income, six income tax brackets, and the corresponding marginal tax rates for each tax bracket (see Appendix A).

The labor income process during the worklife has both permanent and transitory components with uncorrelated and normally distributed shocks as  $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$  and  $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$ . We estimate the deterministic component of the wage rate process  $w_t^i$  along with the variances of the permanent and transitory wage shocks  $N_t^i$  and  $U_t^i$  using the 1975–2017 waves of the PSID. These are computed separately by sex for three education levels: high school dropouts, high school graduates, and those with at least some college (<HS, HS, Coll+).<sup>13</sup> Wages rates are converted into yearly income by assuming a 40-hour workweek and 52 weeks of employment per year. At age 66, the retiree receives a combined income stream from her DC plan and social security benefits, and from age 85 on, payments from the longevity annuity. Results for the six subgroups appear in Figure 1, where Panel A reports the expected income profiles for males, and Panel B for females, by education group. Using 200,000 simulation paths, we re-estimate each of labor income profiles using a Markov-switching model with three income levels, to generate the time-dependent transition matrices for permanent income as well as the age-dependent transitory shocks (see Appendix B).

9 This approach satisfies the Qualified Default Investment Alternative (QDIA) rules as per US Department of Labor regulations (US DOL nd\_a).

10 See 401(k) HelpCenter.com (2017).

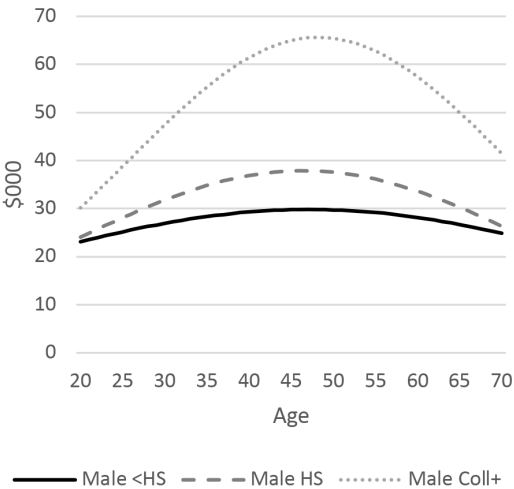
11 The 2019 Secure Act raised the RMD age to 72, and there is an ongoing discussion about whether to raise the age further to age 75; see Tepper (2022).

12 Accordingly, the annual Primary Insurance Amount (or the unreduced social security benefit payment) equals 90 percent of (12 times) the first \$885 of average indexed monthly earnings, plus 32 percent of average indexed monthly earnings over \$885 and through \$5,336, plus 15 percent of average indexed monthly earnings over \$5,336 and through the cap \$10,600. All dollar values are reported in \$2017.

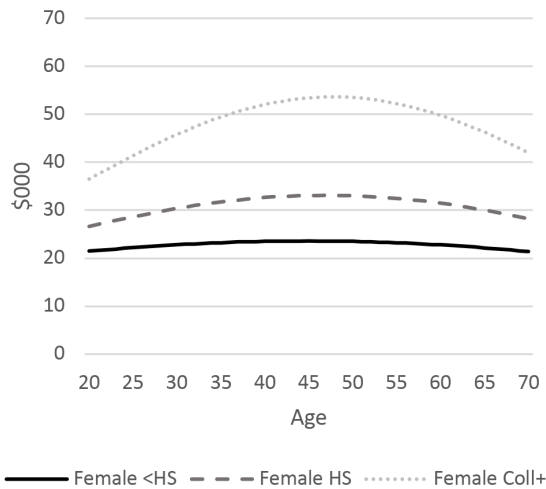
13 Additional details appear in Appendix C.

Figure 1. Estimated average income profiles for females and males by educational level

Panel A. Male expected income profiles



Panel B. Female expected income profiles



Note: The average income profiles are based on wage rate regressions using PSID data (see Appendix B), assuming a 40 hour work-week and 52 weeks of employment per year. Educational groupings are <High School, High School graduate, and at least some college (<HS, HS, Coll+). Source: Authors' calculations.

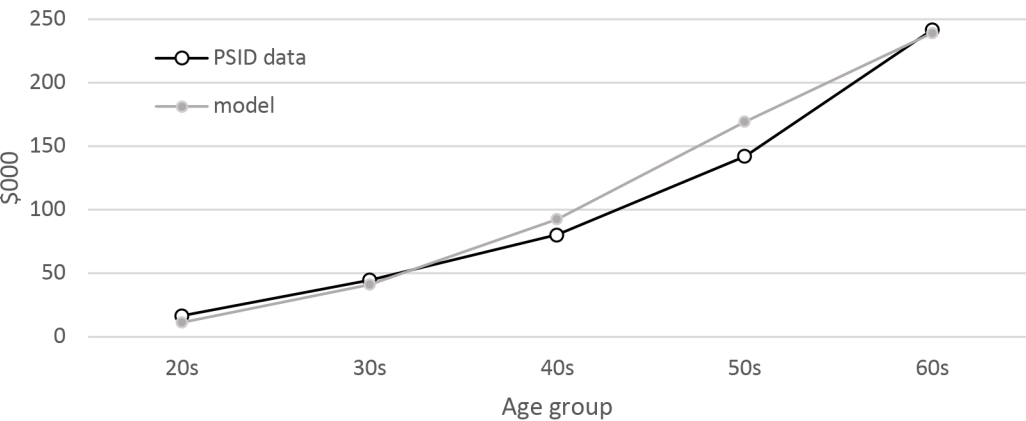
We use dynamic stochastic programming to solve the individual's optimization problem. There are five state variables: wealth, the total value of the individual's DC assets, payments from the longevity income annuity, three income levels, and time.<sup>14</sup> We also compute individual consumption and welfare gains under alternative scenarios using our modeling approach. Values of the preference parameters for the six subgroups are selected so that the model generates DC wealth profiles consistent with empirical evidence. Specifically, we calibrate the model to PSID data 1999-2017 for five age groups (20-29, 30-39, 40-49, 50-59, and 60-69). To generate DC simulated balances, we first

solve the lifecycle model where people claim at age 66 and lack access to longevity income annuities. Then we use the model to generate 200,000 lifecycle simulations weighted for the six subgroups<sup>15</sup> to generate the matching values. We find that a risk aversion coefficient of  $\rho$  of 7,  $\psi$  of 0.35,  $b$  is 1.1, and a time discount rate  $\beta$  of 0.95 are the parameters that closely match simulated model outcomes to empirical evidence on pension assets. Figure 2 displays simulated and empirical data for five age groups, and interestingly, our simulated outcomes are remarkably close to the empirically-observed account values.

14 For discretization, we split the five dimensional state space by using a  $40(X) \times 20(L) \times 15(PA) \times 3(I) \times 76(t)$  grid size. For each grid point we calculate the optimal policy and the value function.

15 To obtain national averages, we aggregate the 200,000 simulated subgroup lifecycle patterns using weights from the National Center on Education Statistics (2016). Specifically, the number of simulations by sex/education are: male <HS 13,000, HS 30,000, Coll+ 57,000; females <HS 11,000, HS 28,000, Coll+ 61,000.

Figure 2. Average simulated versus empirical defined contribution plan asset values (\$000)



Note: The figure compares empirical DC tax-qualified account balances across the US population by age, using PSID data (in \$000), with our life cycle model simulations where workers lack access to DIAs. Model simulations are based on average defined contribution asset levels over 200,000 simulated life cycles of employees. The number of simulations by sex/education are: male <HS 13,000, HS 30,000, Coll+ 57,000; females <HS 11,000, HS 28,000, Coll+ 61,000 as per the National Center on Education Statistics (2016). Source: Authors' calculations.

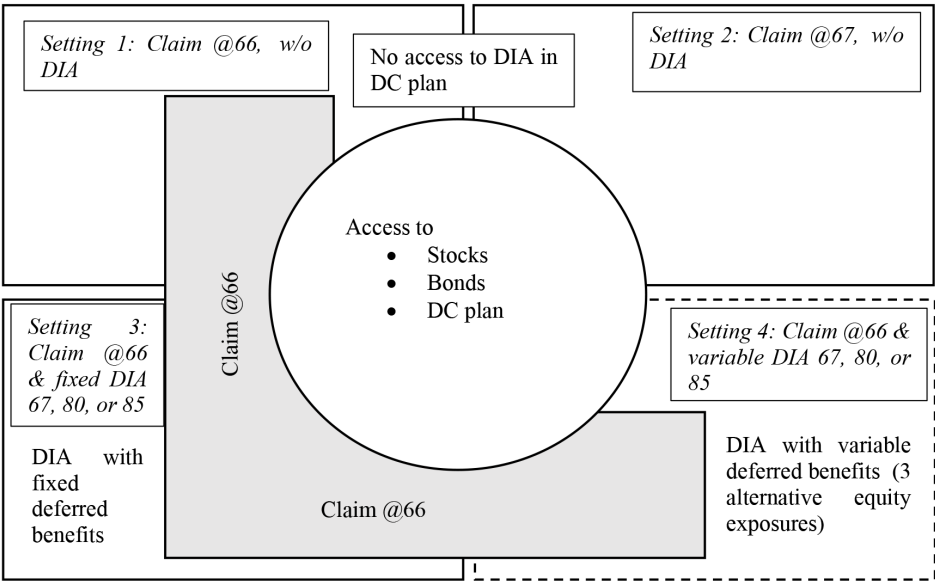
III. Results

We analyze and compare DC wealth and withdrawal profiles, retirement income, and consumption behavior in four different settings: claim @66 w/o DIA; claim @67 w/o DIA; claim @66 w/ fixed DIA 85; and claim @66 w/variable DIA 85). In all four settings, the investor can consume and invest inside and outside her DC plan. In the first and second settings, the retiree lacks access to a DIA. The difference between the first and

second setting is the retirement claiming age: 66 versus 67. In the second setting, the retiree receives no earned income and on social security benefits, so she must withdraw from her retirement plan to cover her consumption. The following year, her social security benefits are 8% higher, from age 67 onward. In setting 3 (4), the retiree has access to a fixed (variable) DIA with payouts starting at age 85; her claiming age remains at 66. Figure 3 gives an overview of the different settings:



Figure 3. Overview of the settings compared



Note: This figure outlines the model settings examined here with respect to two claiming ages (66 or 67) and with or without access to a fixed or variable income annuity deferred to age 67, 80, or 85 held in a qualified defined contribution plan. Setting 4 embeds variable annuities with three patterns of equity exposure: 20%, 50%, and according to a life cycle glide path. Settings outlined in solid lines are currently permitted under QLAC regulations, whereas the cases outlined in dashes are not. Source: Authors' calculations.

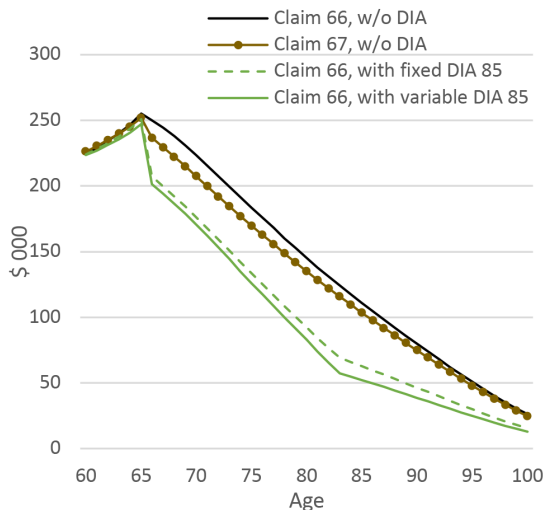
Expected optimal life cycle patterns in retirement

Figure 4 allows a comparison of the expected optimal life cycle patterns for plan assets (Panel A) and withdrawals (Panel B); labor earnings, social security benefits, and DIA payments (Panel C); and consumption (Panel D) based on simulated data for the US population in the four settings for ages 60-100. In all panels, the black line shows all outcomes when the retiree lacks access

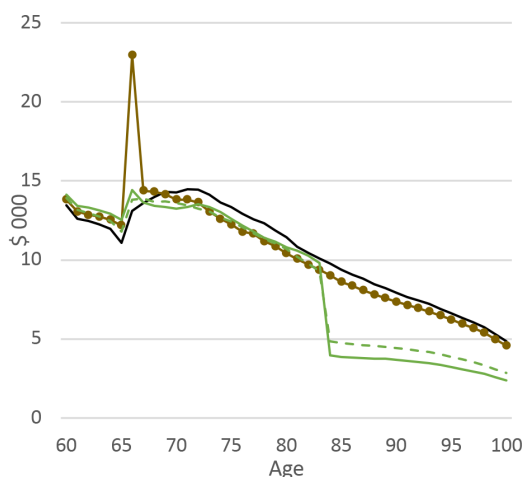
to DIA and retires at age 66. The brown dotted line describes the profiles relevant to the individual who also lacks access to the DIA but defers retirement by one year (to age 67). The green dotted line portrays averages when the individual can purchase a fixed DIA using her DC plan assets up to the allowed limits. The green solid line shows results for the average individual permitted to purchase a variable DIA using her retirement plan assets.

**Figure 4. Life cycle profiles without vs with access to a Deferred Income Annuity (DIA) with fixed or variable payouts**

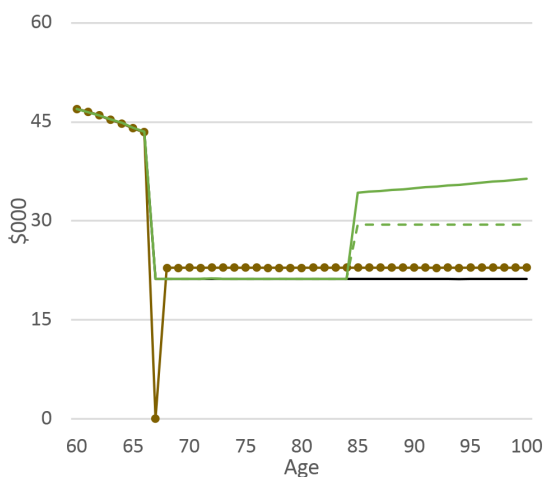
**Panel A. Defined contribution asset values**



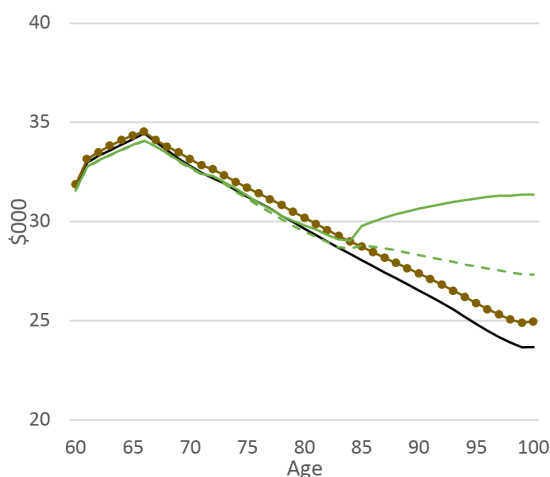
**Panel B. Defined contribution plan withdrawals**



**Panel C. Labor, social security, and annuity income**



**Panel D. Consumption**



Note: These figures show expected values from 200,000 simulated lifecycles by age for US workers having access to tax-qualified defined contribution plans. Panel A shows average DC asset values; Panel B depicts average DC withdrawals; Panel C shows labor, social security, and DIA payouts; and Panel D reports consumption. For additional detail see Figure 2. Source: Authors' calculations

During their work lives, individuals save a portion of their pay in tax-qualified DC plans, building assets (including returns) by age 65 that are worth an average of \$250,000 in all four settings (Panel A). Large differences in retirement plan wealth are not apparent until age 66, when retirees without DIAs who claim social security benefits at 66 must withdraw assets to support their consumption. In the “Claim @67, w/o DIA” setting, the retiree also withdraws money to cover consumption,

with a notable spike in withdrawals averaging \$23,000 (Panel B) due to having no social security benefits or earnings in that year. The decrease in assets observed at age 66 for retirees with access to DIAs is attributable to their purchasing either a fixed or variable DIA, as well as to finance consumption: average withdrawals total approximately \$50,000, of which about \$36,000 is used for the DIA purchase, and \$14,000 to cover consumption costs. Thereafter, withdrawal amounts are similar under

the four settings until age 85. The patterns diverge from age 85 onwards, where the retiree lacking access to DIAs continues withdrawing smoothly until the end of life, whereas retirees with DIAs nearly exhaust their retirement accounts, relying instead on the fixed (variable) DIA benefits for life.<sup>16</sup>

In Panel C, we compare labor, social security, and DIA income in the four settings. The black solid line traces out the drop in labor earnings at age 66, with social security benefits replacing about half of labor income. Deferring claiming by a year implies no labor income at age 67, and social security benefits are then higher by 8% for life due to the delay in claiming. When the retiree has access to a DIA, incomes increase from age 85 onward. This is due to fixed (variable) DIA annual payouts of \$8,200 (\$13,000 and \$15,000 in expectation).

Panel D reports average consumption profiles in the four settings. The lowest average consumption path is observed for the retiree lacking DIA access and claiming at age 66. By comparison, the “Claim @67, w/o DIA” setting provides higher consumption between age 60-100 than the base case: evidently the 8% social security benefit increase more than compensates the retiree for having taken the large withdrawal from her DC plan at 67. The consumption boost experienced in the second setting also rises, the older is the retiree. A retiree with a fixed DIA has virtually the same consumption from age 85 forward, compared to the base case. Thereafter, when the DIA income kicks in, consumption rises substantially and the gap grows with age. A similar pattern applies in the variable DIA case, though the increase in expected consumption begins slightly earlier, but the difference is much higher. For instance, at the age of 90, expected consumption would total \$26,500 with no annuity, \$27,400 if the retiree delayed social security claiming,

\$28,300 with a fixed DIA, and \$30,700 with a variable DIA.

A look at Panels B-D confirms that optimal consumption in retirement can exceed annual income. For example, the retiree who claims at 66 and has access to a fixed annuity would have social security income, DIA-benefits and DC plan withdrawals at age 85 totaling \$34,100. Expenditures for taxes and consumption are lower and total \$31,100. This confirms that the retiree behaving optimally is not always a hand-to-mouth consumer (as in Munnell et al. 2022).

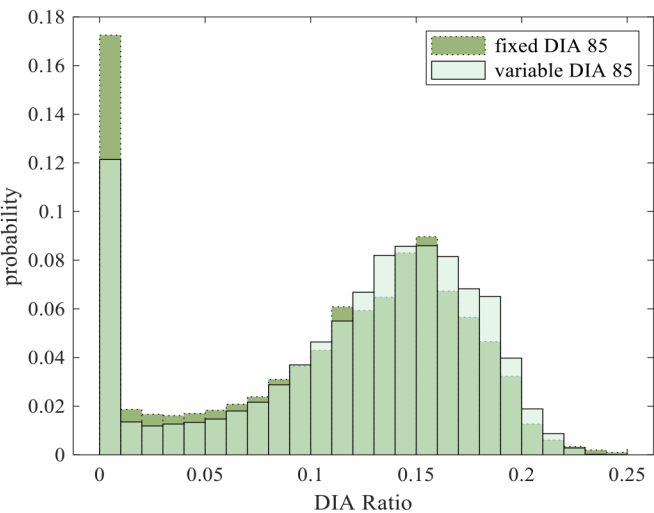
### Optimal demand for DIA at retirement

Let us now consider optimal investments at retirement in the two types of DIAs of interest here. The first has a fixed payout from age 85 onward, which we call the “fixed DIA 85.” The second, which we label the “variable DIA 85,” is invested half in equities and provides a variable payout from age 85. The *DIA Ratio* indicates the optimal percentage of assets in the tax-qualified retirement plan that the retiree will convert to a DIA at age 66. Panel A in Figure 5 compares the distribution of the ratio for the fixed versus the variable DIA; both distributions are based on the 200,000 simulation paths for the US population discussed above (men/women for three education levels). The x-axis in both figures runs from 0% to the maximum value of 25%, where the latter results from the IRS tax qualification requirements for a longevity annuity to count under the RMD rules. Results show that about 84% of the population would be interested in a “fixed DIA 85;” by contrast 88% of the population would favor the “variable DIA 85.” Panel B shows that for a given DIA ratio, the demand for a variable DIA exceeds the demand for a fixed annuity.

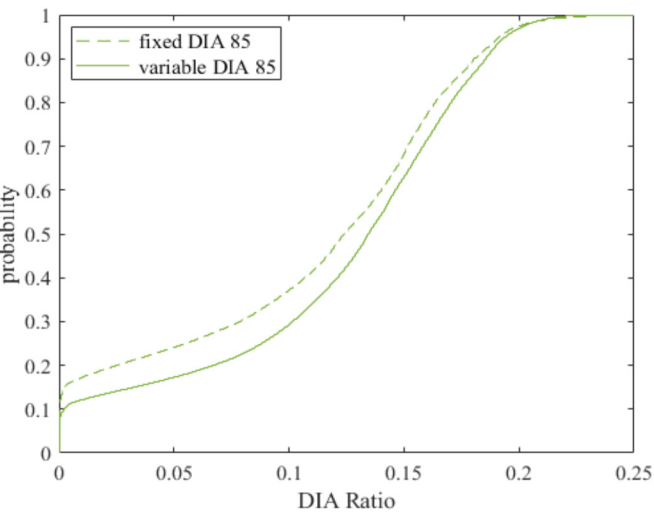
16 This is similar to what Munnell et al. (2022) report, in line with their assumed rule of thumb drawdown plan when the retiree depletes all of her retirement assets by age 85 when she buys the DIA.

Figure 5. Distribution of optimal DIA ratios with fixed or variable deferred annuities

Panel A. Probability distribution



Panel B. Cumulative probabilities



Note: The *DIA Ratio* indicates the optimal percentage of DC plan assets that the retiree converts to a DIA at age 66, payable from age 85. The dark green bars in Panel A indicate the relative frequency of DIA Ratios purchased at age 66, generated from 200,000 simulated lifecycles for US workers having access to a deferred fixed DIA in their defined contribution plans; the light green bars indicate the demand for variable DIAs having 50% equity exposure for the same simulated workers. Panel B shows the corresponding cumulative probability distribution of optimal DIA ratios: the solid line refers to fixed DIAs and the dotted line to variable DIAs. For additional details see Figure 3, settings 3 and 4. Source: Authors' calculations

Table 1 reports the average optimal DIA ratio for the distinct sex and education groups considered when we vary the deferral period of the annuity. We also examine the case of annuities that pay benefits from age 67, and separately, from age 80 onward. The optimal investment in a “fixed DIA 85” for a college educated female (HS, <HS) is 12% (9%, 4%) in expectation. For a college educated male (HS, <HS), the optimal DIA Ratio is higher, at 13% (9%, 6%). The most important reasons for this are twofold: first, the least educated have higher mortality rates, and second, the social security annuity is relatively higher for the lower earners. For the immediate annuity our numbers for the optimal DIA ratio are in line with the 20% default ratio suggested by Munnell et al (2022). Moreover, our results for the DIA payable at age 85 are similar to those reported by Horneff et al. (2020), and Figure 5, Panel B shows that 5% of the population would optimally elect DIA ratios of 20% or more. Accordingly, our optimal life cycle results are compatible with a

substantially lower demand for DIAs than the 20% default rate examined by Munnell et al. (2022) for the same product.

Moreover, for all educational groups, the demand for a variable DIA is 1%-2% greater than for the fixed DIA. We also observe that for shorter deferral periods, the optimal DIA ratio increases, regardless of sex, education, and type of DIA. Moreover, the gap is less for shorter deferral periods, comparing the most and the least educated. For example, the optimal demand for a fixed DIA payable at age 85 is three times higher for the college educated female, compared to the high school dropout. By contrast, the DIA ratio payable at age 67 is only 1.4 times higher for the most versus the least educated. Again, this is because the least educated face higher mortality rates, hence desire earlier payments, and they also receive relatively higher social security replacement rates.

**Table 1. Optimal DIA ratios by sex/education subgroups and alternative deferral ages**

Sex	Education	Fixed DIA			Variable DIA		
		Age 85	Age 80	Age 67	Age 85	Age 80	Age 67
Female	Coll+	0.12	0.18	0.21	0.13	0.20	0.22
	HS	0.09	0.16	0.20	0.10	0.18	0.22
	<HS	0.04	0.08	0.15	0.05	0.11	0.18
Male	Coll+	0.13	0.18	0.21	0.14	0.20	0.22
	HS	0.09	0.15	0.20	0.11	0.18	0.21
	<HS	0.06	0.10	0.17	0.07	0.13	0.20

Note: The DIA Ratio refers to the fraction of the individual's DC plan assets used to purchase the DIA at age 66, for alternative payout start dates. The variable DIA is invested 50% in equities; see text. Source: Authors' calculations.

Naturally, this does not mean that a DIA which starting payments at age 67 will provide a higher utility than a DIA that starts at age 80. Instead, the comparisons assume that retirees in the two different settings have no choice between deferred DIAs with different payout start dates.

#### Welfare gains

Next we compare the utility implications of the four settings in Table 2, where the values illustrate how much more an individual in the "Claim @66, w/o DIA" reference setting would need in her retirement plan to achieve the same welfare as in other settings. All of the numbers in the table are positive, meaning that retirees in the other settings are better off than in they would be if they

claimed at age 66 with no DIA. For example, a college educated woman would require \$38,804 in additional wealth if she lacked access to a fixed DIA 85; \$41,305 if she lacked access to a variable DIA 85 invested half in equity; and \$20,594 if she lacked access to DIA but could claim a year later, at age 67.<sup>17</sup> For college educated men, the welfare gains are, respectively, \$46,870, \$50,207, and \$26,809. Hence, for both educated men and women, access to the variable DIA is more welfare enhancing compared to the case where they claim at 66 and have no DIA. The leisure available in both regimes is identical since the individuals do not work longer, but they wait a year to apply for social security benefits.

**Table 2. Additional cash (\$) the individual requires to achieve the same utility w/o access to a DIA paying out from age 85: Welfare Analysis A (Reference case Setting 1, Figure 3)**

Sex	Education	Claim @66		Claim @67
		Fixed DIA	Variable DIA	w/o DIA
Female	Coll+	38,804	41,305	20,594
	HS	14,528	16,264	11,560
	<HS	3,410	4,295	7,725
Male	Coll+	46,870	50,207	26,809
	HS	16,215	18,635	14,767
	<HS	6,360	7,939	10,641

Note: The reference case in this table is "Claim @66, w/o DIA." The values refer to the additional amount that must be paid into the DC plan that would yield the same utility to the individual who claims her social security benefits at age 66 and has no access to a DIA, versus the three settings indicated. Source: Authors' calculations.

17 This is similar to Munnell et al. (2022) who report that wealthier retirees would have higher welfare gains by using retirement assets to purchase a fixed DIA versus delaying claiming social security benefits.



The same is not true for all educational groups, illustrating the importance of population heterogeneity. For instance, a female high school dropout is financially better off if she claims later. Thus, she would need \$7,725 more assets if she could not claim at 67 instead of age 66. Conversely, if she lacks access to a fixed (variable) DIA, then she would require only \$3,410 (\$4,295) more to make her as well off as claiming at 66. A similar situation arises with the male high school dropouts. In other words, the least educated do relatively better by deferring claiming a year without any annuity, because they have a higher social security replacement rate (not capped) and a higher mortality risk, hence are less interested in deferred annuities of either type.

Next we evaluate how much additional money the retiree in the “Claim @67, w/o DIA” setting would need to have the same welfare as in the “Claim @66, w/ fixed DIA” setting; results using this new reference case and three deferral ages (85, 80, and 67) are provided in Table 3. Here all groups except the high school dropouts

would favor a fixed DIA rather than claiming later. For instance, a college educated woman who could delay claiming but lacks access to the fixed DIA requires an additional \$17,367 in her DC plan to be as well off. The opposite is true for female high school dropouts: they are, on average, \$4,056 worse off if they cannot delay claiming but do have a DIA. A similar pattern applies to men. There are two reasons for this difference. The first is that social security benefits are capped for high earners. Since college educated workers earn more than the social security earnings cap, when they delay claiming, they receive 8% more benefits but only to the cap. Accordingly, the better educated benefit less at the margin compared to a high school dropout earning below the cap. The second reason has to do with different survival probabilities by education: the least educated are likely to die earlier, implying that a DIA payable from age 85 is less attractive for the lower educated, compared to a shorter deferral period.

**Table 3. Additional cash (\$) the individual requires to achieve the same utility w/o access to a DIA paying out from age 85, 80, or 67: Welfare B (Reference case Setting 2, Figure 3)**

Sex	Education	Age 85	Age 80	Age 67
Female	Coll+	17,367	20,989	7,926
	HS	2,832	6,020	1,916
	<HS	-4,056	-2,690	-2,779
Male	Coll+	19,129	21,729	6,449
	HS	1,368	4,104	-0,664
	<HS	-4,021	-2,077	-2,620

Note: The reference case in this table is “Claim 67, w/o DIA.” The values refer to the additional amount that must be paid into the DC plan that would yield the same utility to the individual who claims her social security benefits at age 67 and has no access to a DIA, versus an individual who claims at age 66 and can purchase a fixed DIA with payouts starting at the three deferral ages indicated. Source: Authors’ calculations.

Table 3 also shows that fixed DIAs payable from age 80 onward are also most attractive to men and women of all education levels, versus a fixed DIA payable from age 67. For instance, female (male) college graduates have 2.2-2.6 (2.6-3.0) times higher welfare gains from a deferral age of 80, compared to the DIA paying from age 67. This is true because the DIA payout for the younger deferral age is costly while the deferred product costs little and pays out a much higher benefit to those who live long. Nevertheless, rapidly rising mortality risk beyond age 80 makes the later deferral less appealing. For example, the probability that a male age 66 attains age 80 is 65%, but only 45% for reaching age 85.

Next, in Table 4 we extend the analysis of welfare results by examining three types of variable DIAs embodying stock: a 20% fixed fraction, a 50% fixed fraction, and a

life cycle glide path with the equity share totaling (125–Age/100). As in Table 3, the dollar values represent the additional assets the retiree would need if she held the respective DIA, versus claiming at age 67 with no annuity. We see that giving retirees access to some equities in their variable DIAs increases their welfare gains, compared to having only access to fixed annuities. This is true for all education/sex groups and all deferring ages. For the variable DIA with either 50% equities or a life cycle glide path payable from age 80, educated women can expect an additional welfare gain of 15% compared to the fixed DIA; for men the comparable gain is on the order of 20%. Interestingly, even the smallest equity exposure we study, of 20%, boosts welfare of the high school graduates by more than 20% compared to a fixed deferred annuity.

Table 4. Additional cash (\$) the individual requires to achieve the same utility w/o access to alternative variable DIAs: Welfare Analysis C (Reference case Setting 2, Figure 3)

Sex	Education	Age 85	Age 80	Age 67	Age 85	Age 80	Age 67	Age 85	Age 80	Age 67
Female	Coll+	19,426	23,316	7,526	19,753	24,074	8,053	19,584	23,667	7,507
	HS	3,923	7,545	1,963	4,489	8,425	2,638	4,469	8,273	2,377
	<HS	-3,642	-2,018	-2,345	-3,224	-1,546	-1,887	-3,267	-1,607	-1,966
Male	Coll+	20,862	24,611	4,907	22,316	26,492	7,562	22,381	26,728	9,517
	HS	2,801	5,954	0,774	3,654	7,695	1,641	3,554	7,114	1,626
	<HS	-3,223	-1,045	-1,814	-2,538	-80	-1,066	-2,681	-178	-1,398

Note: The reference case in this table is “Claim 67, w/o DIA.” The values refer to the additional amount that must be paid into the DC plan that would yield the same utility to the individual who claims her social security benefits at age 67 and has no access to a DIA, versus an individual who claims at age 66 and can purchase a variable DIA with different equity exposures and payouts starting at the three deferral ages indicated. Source: Authors’ calculations.

It is also notable that the gains are most positive for the better educated males and females, and negative for the least educated. This is because, for the least educated, delaying claiming social security and using their accumulated savings to bridge their consumption for a year is strongly preferred to an annuity offered by a life insurance company. We also note that the DIA with 20% stocks behaves relatively similarly to the payout structure of a participating annuity. The latter will depend on the overall investment experience of a life insurance company investing in roughly 80% bonds with the remainder in risky assets like stocks.<sup>18</sup> Once again, we note that the annuity deferral to age 80 is preferable to the two other deferral ages considered.

### III. Conclusions

This paper explored the welfare impact of providing access to longevity income annuities inside tax-qualified retirement accounts. We incorporate the social security benefit and tax structure, income taxes, and other institutionally relevant details including required minimum distributions. Our life cycle model recognizes key heterogeneity among the US population in terms of earnings and survival patterns. We extend prior research by comparing the value of purchasing private annuities, versus using funded retirement accounts for bridge financing which permits retirees to receive higher lifelong social security benefits by deferring claiming.

Our results show that using retirement account assets to purchase at least some fixed deferred income annuities is welfare enhancing for all sex/education groups examined. Nevertheless, better educated males and females benefit far more—7 to 11 times more—compared to the least educated. We also find that a

deferral age of 80 is strongly preferred to an immediate annuity, and also to the maximum deferral age of 85 allowed under IRS rules.

We also see that the better educated favor using retirement plan assets to purchase the DIA, versus delaying claiming social security benefits by a year and financing consumption from retirement plan withdrawals. By contrast, lower educated retirees prefer the opposite strategy: they do much better if they delay claiming and use retirement assets to bridge their consumption needs, versus buying the DIA. This is because the least educated have a higher social security replacement rate and a higher mortality risk, whereas the better educated receive relatively lower social security benefits and can anticipate longer lifetimes.

Finally, we also document that providing access to variable deferred annuities with some equity exposure (similar to participating annuities) further enhances retiree wellbeing, compared to having access only to fixed annuities. Regulatory policy stipulates that variable annuities, currently disallowed as QLACs in retirement plans, would contribute to greater wellbeing in retirement. Specifically, a variable DIA payable from age 80 having either a 50% stock fraction or a life cycle glide path provides better educated women an additional welfare gain of 15% compared to the fixed DIA; for similar men, the comparable gain is on the order of 20%. Moreover, even a small equity exposure in the DIA, on the order of 20%, generates around 12% gain for the most educated men and women, and over 20% for high school graduates. Allowing variable DIAs in retirement plan portfolios as qualified longevity annuity contracts would enhance retiree financial wellbeing.

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18 For more detail on participating annuities, see Maurer et al. (2016).

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## Appendix A: Tax treatment in our model

We integrate a US-type progressive tax system in our model to explore the impact of having access to a qualified (tax-sheltered) defined contribution account; all dollar values are in \$2017. The worker pays taxes on labor income and on capital gains from investments in bonds and stocks. During her work life, she invests  $A_t$  in her tax-qualified pension account which reduces taxable income to an annual maximum amount  $D_t = \$18,000$  (additional catch-up contributions of \$6,000 per year are permitted from age 51). Correspondingly, withdrawals  $W_t$  from the tax-qualified account increase taxable income. The worker's taxable income is reduced by a general standardized deduction equal to \$6,350 per year for a single individual. Consequently, taxable income in working age is given by:

$$Y_{t+1}^{tax} = \max[\max(S_t \cdot (R_{t+1} - 1) + B_t \cdot (R_f - 1); 0) + Y_{t+1}(1 - h_t) + W_t - \min(A_t; D_t) - GD; 0] \quad (A1)$$

For social security ( $Y_{t+1}$ ) taxation up to age 66, we use the following rules: when the *combined income*<sup>19</sup> is between \$25,000 and \$34,000 (over \$34,000), 50% (85%) of benefits are taxed.<sup>20</sup> After age 66, we set  $A_t = 0$ , i.e. no further contributions to defined contribution retirement plans occur since the individual stops working.

In line with US rules for federal income taxes, our progressive tax system has seven income tax brackets (IRS 2017). These brackets  $i = 1, \dots, 7$  are defined by a lower and an upper bound of taxable income  $Y_{t+1}^{tax} \in [lb_i, ub_i]$  and determine a marginal tax rate  $r_i^{tax}$ . In the year 2017, the marginal taxes rates for a single person are 10% from \$0 to \$9,325, 15% from \$9,326 to \$37,950, 25% from \$37,951 to \$90,900, 28% from \$90,901 to \$191,650, 33% from \$191,651 to \$416,700, 35% from \$416,701 to \$418,400 and 39.6% above \$418,401 (see IRS 2017). Based on these tax brackets, the dollar amount of taxes payable is given by:<sup>21</sup>

$$\begin{aligned} IT_{t+1}^{tax} = & (Y_{t+1}^{tax} - lb_7) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \cdot r_7^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_6) \cdot 1_{\{lb_7 > Y_{t+1}^{tax} \geq lb_7\}} + (ub_6 - lb_6) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \right) \\ & \cdot r_6^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_5) \cdot 1_{\{lb_6 > Y_{t+1}^{tax} \geq lb_5\}} + (ub_5 - lb_5) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_6\}} \right) \\ & \cdot r_5^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_4) \cdot 1_{\{lb_5 > Y_{t+1}^{tax} \geq lb_4\}} + (ub_4 - lb_4) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_5\}} \right) \\ & \cdot r_4^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_3) \cdot 1_{\{lb_4 > Y_{t+1}^{tax} \geq lb_3\}} + (ub_3 - lb_3) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_4\}} \right) \\ & \cdot r_3^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_2) \cdot 1_{\{lb_3 > Y_{t+1}^{tax} \geq lb_2\}} + (ub_2 - lb_2) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_3\}} \right) \\ & \cdot r_2^{tax} \\ & + \left( (Y_{t+1}^{tax} - lb_1) \cdot 1_{\{lb_2 > Y_{t+1}^{tax} \geq lb_1\}} + (ub_1 - lb_1) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_2\}} \right) \\ & \cdot r_1^{tax} \end{aligned} \quad (A2)$$

19 Combined income is sum of adjusted gross income, nontaxable interest, and half of her social security benefits.

20 See <https://www.ssa.gov/planners/taxes.html>

21 We assume that capital gains are taxed at the same rate as labor income, so we abstract from the possibility that long-term investments may be taxed at a lower rate.

where, for  $A \subseteq X$  the indicator function  $1_A \rightarrow \{0, 1\}$  is defined as:

$$1_A(x) = \begin{cases} 1 & | x \in A \\ 0 & | x \notin A \end{cases} \quad (\text{A3})$$

In line with US regulation, the individual must pay an additional penalty tax of 10% on early withdrawals prior to age 59 1/2 ( $t = 36$ ).

## Appendix B: Wage rate process modeling

We calibrate the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2017 from age 25 to 69. During the work life, the individual's labor income profile has deterministic, permanent, and transitory components. The shocks are uncorrelated and normally distributed according to  $\ln(N_t) \sim N(-0.5\sigma_N^2, \sigma_N^2)$  and  $\ln(U_t) \sim N(-0.5\sigma_U^2, \sigma_U^2)$ . The wage rate values are expressed in \$2017. These are estimated separately by sex and by educational level. The educational groupings are: less than High School (<HS), High School graduate (HS), and those with at least some college (Coll+). Extreme observations below \$5 per hour and above the 99<sup>th</sup> percentile are dropped. We use a second order polynomial in age. The regression function is:

$$\ln(w_{i,y}) = \beta_1 * age_{i,y} + \beta_2 * age_{i,y}^2 + \beta_{waves} * wave \text{ dummies}, \quad (\text{B1})$$

where  $\log(w_{i,y})$  is the natural log of wage at time  $y$  for individual  $i$ ,  $age$  is the age of the individual divided by 100. OLS regression results for the wage rate process equations appear in Table B1.

To estimate the variances of the permanent and transitory components, we follow Carroll and Samwick (CS; 1997) and Hubener et al. (2016). We calculate the difference of the observed log wage and our regression results, and we take the difference of these differences across different lengths of time  $d$ . For individual  $i$ , the residual is:

$$r_{i,d} = \sum_{s=0}^{d-1} (N_{t+s}) + U_{i,t+d} - U_{i,t} \quad (\text{B2})$$

We then regress the  $v_{id} = \overline{r_{i,d}^2}$  on the lengths of time  $d$  between waves and a constant:

$$v_{id} = \beta_1 \cdot d + \beta_2 \cdot 2 + e_{id} \quad (\text{B3})$$

where the variance of the permanent factor  $\sigma_N^2 = \beta_1$  and the  $\sigma_U^2 = \beta_2$  represents the variance of the transitory shocks.

To save calculation time we discretize the CS-process. We simulate the exogenously-determined labor income process as  $Y_{t+1} = f(t) \cdot P_{t+1} \cdot U_{t+1}$  with a deterministic trend  $f(t)$ , a permanent income component  $P_{t+1} = P_t \cdot N_{t+1}$ , and a transitory shock  $U_{t+1}$ . We divide the trajectories into 10 percentile groups and calculate the mean and the standard deviation (transitory shock) for each group. We work with the equivalent discrete Markov-switching process  $Y_{t+1} = I_{t+1}^l \cdot U_{t+1}^{MS}$  with 10 levels ( $l$ ) and a transitory shock for each level  $U_{t+1}^{MS}$ . The  $\Pi_{ij,t} = \text{Prob}(l_{t+1} = i | l_t = j)$  is a time-dependent transition matrix representing the probability of moving from current ( $t$ ) income level  $j$  to income level  $i$  one year later ( $t+1$ ).

**Table B1. Regression results for wage rates**

Coefficient	Male <HS	Male HS	Male Coll+	Female <HS	Female HS	Female Coll+
Age/100	3.268*** (0.108)	6.035*** (0.0478)	9.382*** (0.0673)	1.381*** (0.110)	2.818*** (0.0457)	4.755*** (0.0689)
Age <sup>2</sup> /10000	-3.466*** (0.129)	-6.502*** (0.0611)	-9.717*** (0.0864)	-1.540*** (0.131)	-3.001*** (0.0587)	-4.974*** (0.0894)
Constant	1.893*** (0.0339)	1.502*** (0.0119)	1.187*** (0.0147)	2.119*** (0.0292)	2.106*** (0.0105)	2.113*** (0.0143)
Permanent	0.00922*** (0.000447)	0.0132*** (0.000206)	0.0196*** (0.000302)	0.00811*** (0.000575)	0.0130*** (0.000198)	0.0208*** (0.000346)
Transitory	0.0276*** (0.00121)	0.0304*** (0.000584)	0.0380*** (0.000802)	0.0218*** (0.00151)	0.0266*** (0.000530)	0.0330*** (0.000920)
Observations	28,197	179,577	149,963	21,124	180,952	132,303
R-squared	0.224	0.286	0.309	0.149	0.262	0.260

Notes: Regression results for the natural logarithm of wage rates (in \$2017) are based on data from the Panel Study of Income Dynamics (PSID) for persons age 25-69 in waves 1975-2017. Independent variables include age and age-squared. Robust standard errors in parentheses. \*\*\* indicate the coefficient is significant at the 1% level. Source: Authors' calculations.

## Appendix C: Population mortality tables by education and sex

A great deal of evidence shows that lower-educated individuals have lower life expectancies than their better-educated counterparts. This is relevant to the debate over whether and which workers would benefit from annuitization.

To explore the impact of these difference in mortality rates by educational levels, we follow Krueger et al. (2015) who calculated mortality rates by education and sex ( $M_{sex}^{education}$ ) as below:

$$\begin{aligned}
 M_{male}^{average} &= 0.13M_{male}^{<HS} + 0.3M_{male}^{HS} + 0.57M_{male}^{Coll+} \\
 &= 0.13(M_{male}^{HS} \cdot 1.23) + 0.30M_{male}^{HS} + 0.57(M_{male}^{HS} \cdot 0.94) \\
 &= 0.9957 \cdot M_{male}^{HS}
 \end{aligned} \tag{C1}$$

Next we calculate the mortality for a male with a HS degree as follows:

$$M_{male}^{HS} = \frac{M_{male}^{average}}{0.9957} \tag{C2}$$

Mortality differentials for a male high school dropout or with Coll+ level education is as follows:

$$M_{male}^{<HS} = \frac{M_{male}^{average}}{0.9957} \cdot 1.23 \tag{C3}$$

$$M_{male}^{Coll+} = \frac{M_{male}^{average}}{0.9957} \cdot 0.94 \tag{C5}$$

Analogously, we calculate the following for females with different levels of education:

$$M_{female}^{<HS} = \frac{M_{female}^{average}}{0.9864} \cdot 1.32 \quad (C6)$$

$$M_{female}^{HS} = \frac{M_{female}^{average}}{0.9864} \quad (C7)$$

$$M_{female}^{Coll+} = \frac{M_{female}^{average}}{0.9864} \cdot 0.92 \quad (C8)$$

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