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Household Investment Puzzles and Probability Weighting

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Executive summary

Conventional economic models assume that people confronted with risky choices maximize expected utility, yet in the real world, they often make predictable errors when evaluating risky outcomes. Specifically, people often overweight small probabilities but underweight large probabilities, a phenomenon called "probability weighting." In a laboratory setting, analysts have shown that probability weighting does shape outcomes, but little is known about this phenomenon outside the laboratory. Our research measures how people probability weight in a nationally representative sample of the U.S. population, and then we test how this behavioral bias affects their actual investment decisions.

We measure probability weighting using custom-designed questions based on incentivized lotteries. Consistent with laboratory studies, we find that a majority of people overweight small probabilities and underweight higher probabilities. This phenomenon is called *Inverse-S* weighting. There are also substantial differences across individuals in the degree to which they weight. Next, we test the relationship between probability weighting and household investment choices. As theory predicts, our *Inverse-S* measure is positively associated both with non-participation in equity markets and with individual stock ownership, but negatively associated with mutual fund ownership. Conditional on stock ownership, *Inverse-S* is positively associated with portfolio under-diversification. Robustness tests rule out risk aversion or low probability awareness as alternative interpretations; for instance, we confirm that probability weighting is not driven by a lack of sophistication or a failure to understand portfolio risk.

This draws on Stephen G. Dimmock, Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg (2017). "Probability Weighting and Household Portfolio Choice: Empirical Evidence." Pension Research Council Working Paper.

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Our evidence that probability weighting strongly affects investment behaviors has important implications for how financial service providers should frame investment decisions for savers and retirees, as well as for the design of financial products. For instance, portfolio under-diversification may appear attractive to some, despite offering poor risk-adjusted returns, as we find that probability weighting likely reflects household preferences rather than a misunderstanding of actual risks. This will make it difficult for advisors to stimulate portfolio diversification unless they can directly target these preferences. For example, advisors could remind investors holding a portfolio of only one or two individual stocks that the probability of underperforming the market is high. As an alternative, financial products combining safe (capital guaranteed) and risky (call options on the stock market) components are attractive for investors who overweight small probabilities, and provide an opportunity for financial institutions to design such products.

Highlights

- People often make predictable errors when evaluating risky outcomes, underweighting large probabilities but overweighting small probabilities. This phenomenon is called "probability weighting."
- Our research measures individuals' probability weighting behavior in a nationally representative sample, and then we test how this behavioral bias affects people's actual investment patterns.
- We find that a majority of people overweight small probabilities and underweight higher probabilities, though there are substantial differences in the degree to which this happens.
- People who probability weight fail to participate in the stock market, but if they do, they fail to diversify.
- Financial advisors would do well to be aware of how probability weighting discourages diversification of investment portfolios.

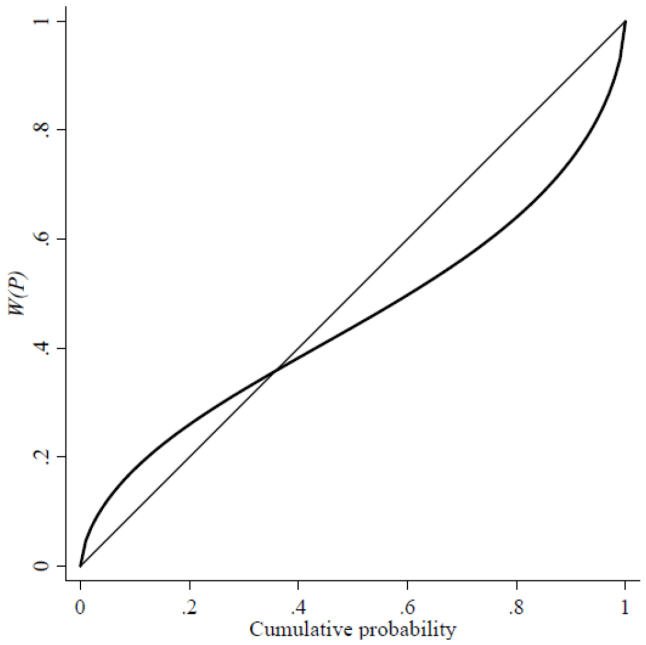
Introduction

Past research has confirmed that many U.S. households do not invest in the equity market, and when they do, they tend to be over-concentrated in one or a few stocks. Our research explores whether a prominent behavioral bias, termed *probability weighting*, can help explain these investment puzzles.

Probability weighting arises when individuals overweight small probabilities but underweight large probabilities. Figure 1 shows an example of probability weighting; for small probabilities the weighted probabilities exceed the actual probabilities, for larger probabilities the reverse is true. Behavioral models of asset allocation and investment decisions based on probability weighting offer some clear predictions not yet tested in prior empirical studies.[1] First, overweighting of extreme events leads to under-diversification and demand for lottery stocks with positively skewed returns (Polkovnichenko, 2005; Barberis and Huang, 2008). An investor who overweights small probability extreme gains would prefer to hold a small number of individual stocks that offer a shot at becoming rich (hoping to identify the next Apple or Google), versus an index fund. Furthermore, probability weighting also makes investment in a broadly-diversified stock market portfolio unattractive (De Giorgi and Legg, 2012).

[1] Our analytical framework follows from several theoretical studies on portfolio choice behavior including Hu and Scott (2007), Barberis and Huang (2008), De Giorgi and Legg (2012), and Bernard et al. (2015).

Figure 1. Probability weighting example



This figure shows actual probabilities on the x-axis, and the transformed probabilities on the y-axis.

In a laboratory setting, studies have shown that probability weighting does shape people's decisions, but little is known about this phenomenon outside the laboratory. Our research measures individuals' degree of probability weighting in a nationally representative sample of the U.S. population, by building and implementing a module in the American Life Panel survey using custom-designed questions and incentivized lotteries. Next, we test how this behavioral bias affects people's actual investment patterns.

Consistent with laboratory studies, we find that a majority of people overweight small probabilities and underweight higher probabilities (*Inverse*-S weighting); there are also substantial differences across individuals in the degree to which this occurs. Next, we test the relationship between probability weighting and household investment choices. As theory predicts, our *Inverse*-S measure is positively associated both with nonparticipation in equity markets and with individual stock ownership, but negatively associated with mutual fund ownership. Conditional on stock ownership, *Inverse*-S is positively associated with portfolio under-diversification. Robustness tests rule out risk aversion or low probability sophistication as alternative explanations; for instance, we confirm that probability weighting is not driven by a lack of sophistication or a failure to understand portfolio risk.

In sum, probability weighting is associated with two well-documented anomalies in household portfolio choice: non-participation in equity markets and portfolio under-diversification among those who do participate. Our evidence that probability weighting strongly affects investment behaviors has important implications for how financial service providers should frame investment decisions for savers and retirees, as well as for the design of financial products. For instance, portfolio under-diversification may appear attractive to some, despite offering poor risk-adjusted returns, as we find that probability weighting likely reflects household preferences rather than a misunderstanding of actual risks. This will make it difficult for advisors to stimulate portfolio diversification unless they can directly target these preferences. For example, advisors could remind investors holding a portfolio of only one or two individual stocks that the probability of underperforming the market

is high. As an alternative, financial products combining safe (capital guaranteed) and risky (call options on the stock market) components are attractive for investors who overweight small probabilities, and provide an opportunity for financial institutions to design such products.

The behavioral context

Kahneman and Tversky (1979) proposed a behavioral model of how people make decisions under risk often used to assess violations of the conventional economic expected utility model. There are three key components of what they called *prospect theory*: framing, loss aversion and probability weighting. In the present research, we focus on *probability weighting,* which has been documented in laboratory studies to be an important feature of individual decision making. Yet little empirical evidence is available about the effects of probability weighting on financial decision making (for a review see Barberis, 2013). Our research evaluates the consequences of probability weighting for household portfolio allocation.

Conventional portfolio choice models predict that virtually all households will participate in the equity market (Merton, 1969), yet in fact this is not the case. One explanation for nonparticipation is that people subject to probability weighting overweight extreme outcomes, both good and bad. This overweighting of extreme bad outcomes exacerbates the effects of risk aversion,[2] resulting in nonparticipation.

Standard portfolio choice models also counterfactually predict that households will hold well-diversified portfolios, whereas numerous studies show that many households own highly undiversified portfolios, with large positions in just one or a few individual stocks.[3] Probability weighting increases sensitivity to skewness of returns, and in view of the negative skewness of the aggregate stock market (Albuquerque, 2012), probability weighting makes owning a well-diversified equity portfolio less attractive.[4] In contrast, probability weighting makes owning an undiversified portfolio of positively skewed securities (such as individual stocks) more attractive.[5]

A testable implication of probability weighting for household investment behavior is that it makes holding a diversified equity portfolio, such as a mutual fund, unattractive. Instead, people with high *Inverse-S* will prefer to either avoid equities completely or hold an undiversified portfolio with a small number of individual stocks. Next, we describe how we test these hypotheses empirically.

Methodology

We developed and implemented a simple elicitation method to measure probability weighting suitable for an internet-based survey of the broader population, adapting and extending the methodology of Wakker and Deneffe (1996) and Abdellaoui (2000). The module elicits certainty equivalents for a series of binary lotteries. The probabilities of winning the lotteries varies from small to large, allowing us to obtain a measure of each individual respondent's probability weighting behavior which we term *Inverse-S*.

Our survey module was fielded in the RAND American Life Panel (ALP) where subjects were provided real incentives randomly rewarded, a standard approach to increase the reliability of survey responses.[6] The ALP consists of several thousand households that regularly answer internet surveys; the ALP sample is quite comparable to the overall U.S. population. Households lacking internet access at the recruiting stage were provided with a laptop and wireless service to limit selection biases (for more see https://alpdata.rand.org/).

[2] See for instance Segal and Spivak (1990), Epstein and Zin (1990), and Chapman and Polkovnichenko (2011).

[3] See Blume and Friend (1975), Kelly (1995), Calvet, Campbell, and Sodini (2007), Goetzmann and Kumar (2008), and Kumar (2009).

[4] See Polkovnichenko (2005), Chapman and Polkovnichenko (2011), De Giorgi and Legg (2012), and He, Kouwenberg, and Zhou (2017).

[5] See Shefrin and Statman (2000), Polkovnichenko (2005), Barberis and Huang (2008), and Jin and Zhou (2008).

[6] For more on the ALP see Dimmock et al. (2016).

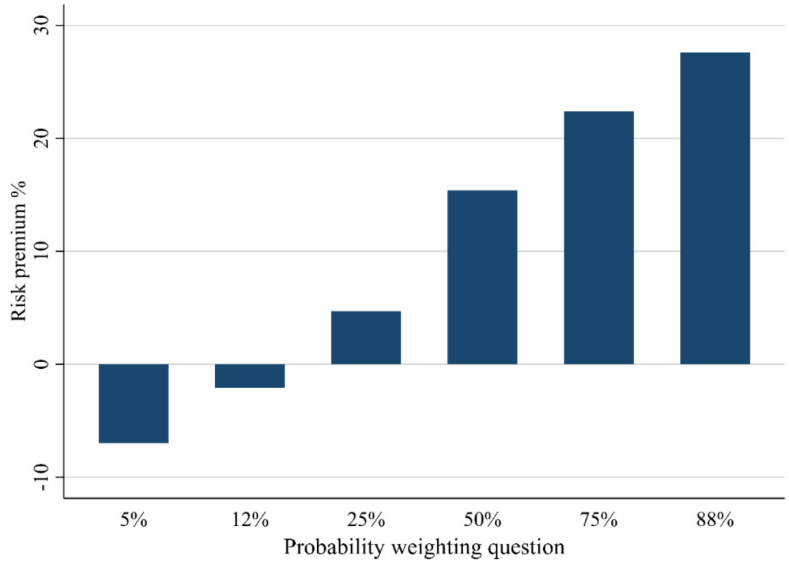
Our ALP module also gathered additional data on respondents collected in earlier modules on their stock market participation and portfolio diversification choices. We also measured respondents' degree of risk aversion, as well as respondents' subjective probabilities of stock market crashes and early death. Next, we tested our hypotheses about probability weighting using multiple regression techniques that control for socio-demographic factors and other behavioral factors that could influence investment decisions. Survey weights were applied so our results should be representative of the U.S. population.

Results

Our results show that, on average, people display *Inverse*-S-shaped probability weighting in their preferences, with substantial differences across individuals. Thus, when the probability of winning a lottery was only 5%, our subjects demanded a certainty equivalent on average that was larger than the expected value of the lottery. In contrast, when the probability of winning a lottery was higher (e.g., 50%), subjects on average would accept a certainty equivalent smaller than the expected value of the lottery.

Figure 2 illustrates this pattern in our own dataset, displaying the average risk premia for a simple binary lottery with prizes of $42 and $6, with the chance of winning $42 varying from 5% to 88% across six questions. For small probabilities (5% and 12%), we find that people are willing to pay more than the expected value on average to take their chance on the lottery, but for high probabilities, they become very risk averse, demanding large positive risk premiums. The real-world analog of such choices is to see the same person buying both lottery tickets and insurance, even though both have negative expected values, and yet the same person might demand a premium to work in a risky occupation.

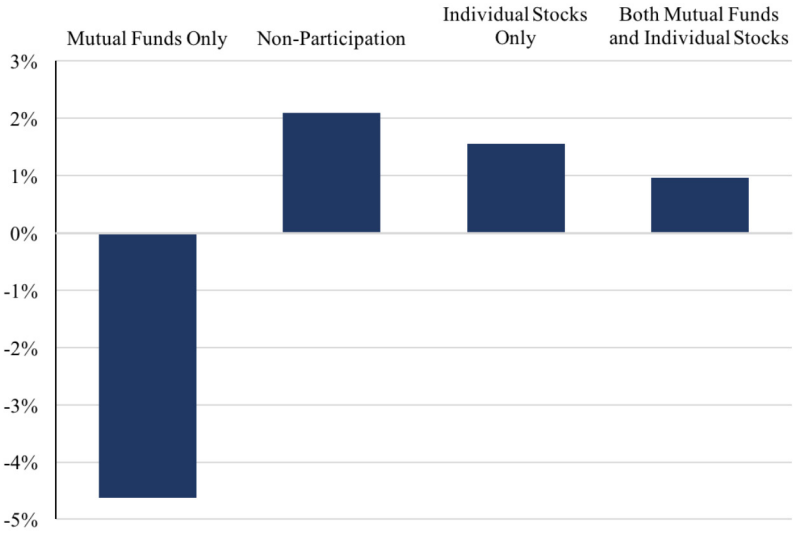
Figure 2. Average risk premia (%) for the six probability weighting questions shown to ALP respondents



This figure shows the average risk premium in percentages (on the y-axis), for each of the six probability weighting questions (with the six probabilities of winning on the x-axis).

Having determined each person's probability weighting measure, we then test whether this explains people's investment choices using a regression model with four outcomes: non-participation in equity markets, investing in mutual funds only, investing in individual stocks only, and investing in both mutual funds and individual stocks. As shown in Figure 3, we find that higher *Inverse-S* is positively associated with nonparticipation and ownership of individual stocks, and thus negatively associated with owning only mutual funds. People in the top quintile of the *Inverse-S* distribution are almost 5 percentage points less likely to have mutual funds only, a large increase, given that on average only 8.2% of the people in the sample own mutual funds only. Instead, these respondents are 2 percentage points more likely to not participate, 1 percentage point more likely to own both mutual funds and individual stocks, and 1.5 percentage point more likely to own individual stocks only. These results are consistent with the implications of probability weighting, but differ from the implications of other features of preferences such as risk aversion, because the subjects choose either the least risky choice (nonparticipation) or the riskiest choice (an undiversified portfolio).

Figure 3. Difference in the probability of holding an asset class between people in the top versus bottom quintile of the *Inverse-S* distribution



For those holding equity, we measure the fraction of equity holdings allocated to individual stocks, which Calvet, Campbell, and Sodini (2007) showed was a good proxy for portfolio under-diversification. Here we find that a one-standard deviation increase in *Inverse-S* implies a 12.8 percentage point increase in the fraction of the portfolio allocated to individual stocks (28.4% relative to the baseline rate of 45.0 percentage points).

Robustness

These results continue to hold when controlling for differences in age, income, financial assets, education, marital status, number of household members, employment, optimism, trust, numeracy, financial literacy, and risk aversion. We also found similar results excluding subjects who answered the elicitation questions unusually quickly or who made multiple errors on the

check questions. Most importantly, we evaluated whether probability weighting reflected preferences versus unintentional errors by the survey respondents, and we conclude that probability weighting reflects preferences rather than unintentional errors.

Conclusions and implications

Our research finds that probability weighting explains household investment decisions that seemingly violate utility-maximizing behavior. It is the first study to show a direct relation between elicited probability weighting preferences and actual household portfolio decisions.[7] In particular, we show that probability weighting can explain both nonparticipation in equity markets and portfolio under-diversification among investors. That is, we demonstrate that people who overweight extreme events avoid investing in the stock market, but those who do invest tend to hold portfolios that are under-diversified.

Further, our research has implications for financial education and retirement plan defaults. Although probability weighting is related to people's preferences and thus difficult to change, it might be possible to alter how financial decisions are framed for savers and retirees, as how financial products are structured. For example, people have a tendency to focus on salient extreme events such as a particular individual stock doubling in price within a matter of months, or the story of a person who saved a great deal but unfortunately passed away soon after retirement. Providing better information about the distribution of long-term expected outcomes may help people frame these financial decisions in a way that encourages better decisions. For example, presenting investors with evidence about the positive average returns from holding a diversified stock market fund could help people diversify better. Advisors could also remind investors holding a portfolio of only one or two individual stocks that the probability of underperforming the market is high. If probability weighting is a preference with adverse consequences for financial outcomes, this would also imply that financial education programs could teach people to better recognize the effects of probability weighting on their savings and investment decisions. Accordingly, our research has implications for how financial institutions might want to design financial products, with the potential to greatly enhance Americans' retirement well-being.

[7] Polkovnichenko (2005) uses actual stock return data to obtain the numerical results for his calibrated model. Rieger (2012) and Erner, Klos, and Langer (2013) relate elicited probability weighting to hypothetical financial decisions about structured products in laboratory experiments using university students. By contrast, we relate elicited preferences to real financial decisions in the field.

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