Executive Summary

Financially inexperienced consumers with self-directed retirement accounts may fail to understand investment and longevity risk. This problem can be rectified if defined contribution plans include variable deferred annuities with lifetime income guarantees and investment downside protection. Our paper evaluates lifecycle consumption and portfolio allocation patterns given Guaranteed Minimum Withdrawal Benefit (GMWB) variable annuities, a rapidly-growing financial innovation in the recent past. A key feature of these products is that they provide access to equity investments with downside protection, hedging of longevity risk, and partially-refundable premiums.

We incorporate fairly-priced GMWBs into the investment opportunity set of a utility-maximizing investor facing an uncertain lifetime, risky labor income and stochastic equity returns, who must select a dynamic path for consumption and portfolio allocation across risky stocks, bonds and GMWBs. We incorporate individual risk-aversion, borrowing constraints, capital market volatility, and other background risks. Because the model is highly realistic, analytical solutions do not exist, so we solve it with efficient numerical procedures.

We show that many consumers will optimally purchase variable annuities prior to retirement because of their flexibility and access to the stock market; also some will take cash withdrawals prior to retirement. For the range of households we examine, welfare gains of up to 4% result from access to the variable annuity/GMWB (compared to no access).
Introduction
Defined contribution pensions are a rapidly-growing form of retirement saving. Yet participants in such self-directed pension plans often fail to understand the risks associated with their investment and spending decisions, exposing them to the risk of potential retirement shortfalls. In particular, financially inexperienced consumers who must manage their self-directed retirement accounts may fail to understand investment and longevity risk. This problem can be rectified if defined contribution plans include variable annuities (VAs) with lifetime income guarantees and investment downside protection. In particular, our recent study (Horneff, Maurer, Mitchell, and Rogalla 2015) shows how such VAs with guarantees can be used to enhance retirement security in the context of a lifecycle model.

Our research posits that many households would likely benefit from incorporating income and return guarantees into their defined contribution pension plans. Specifically, these products offer lifelong benefit payments during retirement, as well as protection of accumulated assets from downside market shocks. Such guarantees are offered by insurers in the form of investment-linked variable annuities, though relatively little is known about how to integrate these products into the pension context. This paper shows how such variable annuities with guarantees can be used to enhance retirement security in the context of a lifecycle model.

The Variable Annuity Marketplace and GMWBs
The most popular variable annuity product, examined in this paper, is a deferred variable annuity with a Guaranteed Minimum Withdrawal Benefit rider. These constitute almost 80% of recent variable annuity sales; as of 2010, some $1.5 trillion of assets were invested in VA contracts in the U.S. By contrast, assets invested in fixed annuities were only $660 billion. In the same year, the flow of annuity sales in the United States totaled $210 billion, two-thirds of which were variable annuities.

VA/GMWBs (or GMWBs for short) are insurance products with both investment and income components. During the accumulation phase, the policyholder pays premiums to a life insurer, which (after expenses) are invested in mutual fund-style sub-accounts. The GMWB policyholder may elect to take back her entire premium in small portions (i.e., a “money-back” guarantee) over a certain time frame, regardless of the actual investment performance of her underlying portfolio. Typically, the consumer may withdraw a maximum annual percentage of her premium until it is completely recouped. Any remaining capital at the end of the deferral period can be converted either into a lifelong annuity or paid to the policyholder in the form of a lump sum.

Because of the withdrawal option, premiums are at least in part refundable, so that GMWBs provide some liquidity which can help overcome consumer reluctance to voluntary annuitize retirement wealth. Then at retirement, the retiree has the possibility of converting her accumulated assets into a lifelong income stream while providing access to upside asset returns. In this way, the GMWB offers access to equity investments, downside protection against market risk, and the possibility of hedging longevity risk via annuitization. For this reason, this product offers access to equity investments with downside protection, hedging of longevity risk, and partially-refundable premiums.

Prior Work
Previous research on variable annuities has taken two approaches. First, some authors have investigated how to price the complex option features embedded in these contracts. Second, the dynamic portfolio choice literature has examined household demand for life annuities and their welfare implications. Though some studies examine immediate and fixed annuities, few have considered variable annuities with deferred benefits in a realistically-calibrated lifecycle portfolio choice model. Nevertheless, no prior work has incorporated the key guarantee features of variable annuities in line with those offered in the market.

Modeling the Product
We describe the single premium deferred variable annuity with a GMWB rider as a contract between a life insurer and a consumer who must pay the firm an initial amount $A$ at time $t = 1$ when signing the contract. The insurance company commits to paying a fixed lifetime benefit stream to the annuitant starting at time $K$ (the end of the deferral period).

2. Among the many studies on annuity pricing see Bauer et al. (2008), Milevsky and Posner (2001), Milevsky and Salisbury (2008), and Ulm (2006). For some prior studies on variable annuities see Chai et al. (2011), Horneff et al. (2007; 2009; 2010; and 2015), Maurer et al. (2013), Milevsky and Posner (2001), and Steinorth and Mitchell (2014).
The firm then invests the premium into a Fund Account $F$ which earns an uncertain growth rate $R_{t+1}$ from time $t$ to $t+1$ and from which the annual fees are deducted. A second account, the Guarantee Account, keeps track of the total guaranteed withdrawal amount still available to the annuitant.

The policyholder may have some control over the risk-return profile of her investments in the Fund Account, by selecting from a menu of mutual funds (e.g., equity, fixed income, real estate). As with all life-contingent annuity products, if the policyholder dies during the deferral period, any remaining cash value in his Fund Account will be transferred to the insurer. Yet in contrast to traditional deferred life annuities, as long as the annuity holder is alive, she may request a return of premium paid to the insurance company, within some limits.

When the insurer selling the GMWB is at risk under this contract, it takes a short position on a (complex) option. Accordingly, it must levy an appropriate risk charge. For instance, if the Fund Account were to be depleted during the deferral period, or, if at the end of the deferral period the remaining Guarantee Account were to exceed the Fund Account, the insurer must pay the shortfall using own resources. The risk charges for the complex income/investment guarantees inside the variable annuities allow for the possibility of periodic withdrawals (within some limits).

To price the guarantees, we use techniques from options pricing theory. Using this approach, we derive the insurer’s annual risk charge expressed as a percentage of the Fund Account. Clearly the fee depends on the policyholder’s age when the contract is signed (the deferral period is assumed to end at age 65), as well on her asset allocation within the product. Our pricing approach assumes that the participant’s Fund Account is fully invested in equities, since that allocation maximizes the value of her guarantee inside the GMWB. The risk charge must rise with the purchaser’s age because it is paid annually instead of as a one-time lump sum; hence the younger buyer pays the charge over more years. Also, assuming the insurer can pool mortality risk, when a policyholder dies, remaining wealth in her Guarantee and Fund Accounts transfers to the insurer. This generates the well-known mortality credit due to mortality risk pooling, which the insurance company incorporates in calculating the product risk charge.

**Figure 1: Annual Risk Charges of Single Premium GMWBs at Alternative Purchase Ages**

![Graph showing annual risk charges in basis points (bps) of the current Fund Account value. The Fund Account is assumed to be fully invested in equities with a volatility of 18%, the risk-free rate is 2%, and the deferral period ends at age 65. The solid (dashed) line represents the situation when the risk charge is calculated with (without) mortality risk pooling. Source: Horneff, Maurer, Mitchell, and Rogalla (2015).]
As is clear from Figure 1, an insurer’s annual risk charge for the GMWB must rise with age. The solid line reflects the fact that a policyholder who purchases the contract at age 40 must pay an annual yearly fee of 26 basis points (bps) of her Fund Account per year until the deferral period of age 65. If the buyer were instead age 50, her yearly fee would rise to 64 bps (assuming the same deferral period). Conversely, someone who purchased the same policy at age 64 would pay 1,080 bps, but for only a single year.

By contrast, the dashed line in Figure 1 illustrates the risk charge to cover a death benefit, in the event that one is provided. A life-contingent GMWB purchased at age 40, for example, would involve a risk charge of 26 bps versus 72 bps for a product that included a death benefit. At age 60, the annual risk charges would be 257 bps versus 448 bps.

The Lifecycle Context

Our paper extends prior research by incorporating such fairly-priced variable annuities with guarantees into the investment opportunity set of a utility-maximizing investor who faces an uncertain lifetime, risky labor income, and stochastic equity returns. Building on and extending our prior work, we next introduce a dynamic consumption and portfolio choice for a utility-maximizing investor over the lifecycle. We use this realistic calibrated lifecycle framework to generate the optimal consumption and portfolio allocation across risky stocks, bonds, and annuities of the sort of interest here.

The model incorporates individual risk aversion, borrowing constraints, capital market volatility, and other background risks. In a base case, we assume that the variable annuities are purchased in non-qualified plans, that is, with contributions from after-tax income. The base case also omits labor income risk. Sensitivity analyses show how demand for such products differs in the context of a tax-qualified retirement plan and with labor uninsurable income uncertainty. Additional robustness checks include alternative valuations for risk aversion, product costs, investment options allowed, bequest preferences, levels of Social Security benefits, and interest rate environments. Finally, we assess the welfare implications of having access to this innovative retirement financial product. Because the model structure and calibrated parameters are highly realistic, analytical solutions do not exist; accordingly we solve this realistic lifecycle model with efficient numerical procedures using parallel-computing on a high-performance cluster technology.

Results

For calibration of the base case parameters, we use standard values in the literature; sensitivity analysis examines results under other sets of parameters. Under our base case scenario, investors will optimally purchase variable annuities prior to retirement because of their flexibility and access to the stock market. Moreover, many consumers will also adjust their portfolios and consumption streams along the way by taking cash withdrawals from the products. Overall, policyholders exercise the product’s flexibility by taking withdrawals and dynamically adjusting their portfolios and consumption streams over time.

Figure 2 displays results for the base case. The top panel reports the paths of expected consumption, labor income, wealth in liquid assets (stock, bonds) and GMWBs, along with annuity purchases over time and withdrawals from existing GMWB accounts. The GMWB value is the greater of either the Fund or the Guarantee Account prior to age 65, and thereafter the (actuarial) present value of the lifetime annuity payments. We define financial wealth as the sum of stocks, bonds, and GMWB values.

In this setting, at age 40, the individual optimally allocates a substantial portion of her financial wealth to the GMWB, about $48,000 (or 42% of total wealth). The value of the Fund Account continues to rise during her worklife, peaking at age 64 when it amounts to about $210,000 in expectation. At age 65, she takes a lump-sum of about $17,000 (about 7.4%) of her GMWB value, which is reflected in the sharp increase in liquid wealth. All remaining GMWB assets are converted into a lifelong annuity paying yearly fixed benefits of about $14,000 (or 49%) of her last labor income. Since no further annuity purchases after age 65 are possible in our model, the present value of the annuitized financial wealth declines with age during retirement.

3. A basis point is equal to 0.01%
Figure 2: Expected Lifecycle Profiles in the GMWB Environment

Notes: The top panel shows the development of labor/pension income, liquid and GMWB wealth, GMWB annuity income, and optimal consumption as explained in the text. The bottom panel displays the development of the fund and the guarantee account, as well as the optimal purchases and withdrawals from the GMWB account. Expected values (in $000) based on 10,000 simulated lifecycles using the base case calibration: risk aversion $\rho=5$; time preference $\beta=0.96$; no bequest motive ($b=0$); initial liquid wealth (labor income) of $120,000 ($29,600 p.a.) at age 40; no labor income risk ($\sigma^2=\sigma^2_l=0$); retirement age: 65; pension replacement rate 73.6% no taxes; risk-free interest rate 2%; mean stock return 6%; stock return volatility 18%. Source: Horneff, Maurer, Mitchell, and Rogalla (2015).
Whereas other studies have predicted that consumers will wait to buy deferred annuities late in life, our results indicate that investors optimally purchase measurable amounts of GMWBs well before retirement because of their flexibility and access to the stock market. This finding is consistent with empirical evidence of the growth in variable annuity demand over time. Moreover, and consistent with observed behavior, differences across cash out and annuitization patterns result from variations in realized equity market returns and labor income trajectories.

The bottom panel of Figure 2 shows average annuity purchases (black bars) and withdrawals (clear bars) from existing accounts, as well as the values of the Guarantee (solid line) and Fund Accounts (dashed line). Account values correspond to the left axis, while purchases and withdrawals refer to the right axis. Focusing first on purchases and withdrawals, we note that when a household buys additional annuities, its corresponding withdrawals amount to zero and vice versa. Purchases (withdrawals) depicted in the figure are generated by averaging over the 10,000 simulated realizations; for this reason, at any given age, some households will purchase new annuities, while others will withdraw funds.

As noted above, at age 40, individuals on average devote a substantial amount of their financial wealth (42%) to GMWBs which are relatively inexpensive due to the low annual guarantee risk charge (about 26 bps). This high initial level of annuity purchases, combined with rising fees for additional purchases, produces negligible additional GMWB purchases until just before retirement. Then at age 64, some individuals take advantage of the final annuitization opportunity and shift a small amount of their liquid wealth into the annuity product despite its relatively high fee (around 14.6%). Other policyholders find it optimal to take small withdrawals from the GMWB Fund Account through age 64. At age 65, unlimited withdrawals are permitted and, on average, people withdraw about $17,000.

The individual’s optimal behavior with respect to withdrawing funds from existing GMWBs as well as purchasing additional GMWBs is complex, as we show in our published paper (Horneff, Maurer, Mitchell, and Rogalla 2015). Her choices depend on the interactions between stock market returns and their impact on the Fund Account, between the Fund and the Guarantee Account values, and the age-dependent fees for the GMWB rider. We also analyze how GMWB access affects policyholders’ optimal lifetime consumption by computing individuals’ average consumption per period with and without access to the GMWB annuity product. Prior to retirement, it appears that consumption differences are rather small. By contrast, in retirement people with GMWB access benefit from the lifelong income stream provided by their annuity. So until around age 80, they can afford to consume annually some $1,000 (3%) more than their counterparts in the non-GMWB world. Later in the lifecycle, this difference in annual income increases are even larger.

To evaluate the total welfare gain associated with the discussed extension of consumption opportunities over the complete lifecycle, we computed the individual’s certainty equivalent wealth at age 40. We then calculated the relative change in certainty equivalent wealth when moving from a world without to a world with GMWBs. For our base case, this welfare gain amounts to 1.7%, a result not too dissimilar from one where the consumer has a bequest motive. Very risk averse individuals are even better off than in the base case (2.5% vs. 1.7%). The consumer exposed to both labor income and capital market shocks values the access to GMWBs substantially more (about 4.3%) than her counterpart who only faces equity risk. In other words, the guarantee and liquidity features, as well as the access to the mortality credit in this investment-linked deferred annuity, make GMWBs very attractive to the consumers examined here.

We also conducted three policy experiments. In the first, we find that people purchase more GMWBs and cash out less, when Social Security and private defined benefit pension benefits are lower. Second, we show that having GMWBs available in a tax-qualified retirement account enhances their popularity; compared to the base case, the addition of taxes induces individuals to purchase more equity and hold less in their GMWB accounts. Moreover, the cash out ratio at retirement is substantially lower than in the base case, as large lump sum withdrawals will be taxed immediately at a high rate due to tax progressivity. Finally, if a GMWB requires deferring the payout until age 85 (in the spirit of pure longevity insurance), retirees cash out more to finance consumption but they still enjoy a welfare gain from access to the product.

Implications and Relevance

Our paper develops a lifecycle consumption and portfolio choice model for an individual who — in addition to stocks and bonds — can gradually purchase fairly-priced deferred variable annuities (GMWBs). Prior to retirement, these
offer access to the stock market with investment downside protection and minimum withdrawal benefits. At retirement, they allow the policyholder to completely cash out the accumulated account or convert it into a fixed lifetime income stream. Such variable annuities have been one of the most rapidly growing financial products over the last decades.

The particular attraction of guarantees in these products is that they offer access to equity investments, downside protection against market risk, and the possibility of hedging longevity risk via annuitization. Our results should be of interest to financial advisers and plan sponsors seeking to enhance employees’ retirement security, as well as to Social Security which is increasingly being asked to take on longevity risk due to the decline of defined benefit plans. Because of the withdrawal option embedded in these products, premiums are at least in part refundable, and this partial liquidity can help overcome consumer reluctance to voluntarily annuitize their wealth.

Compared to an environment without GMWBs, these products contribute to enhanced lifetime utility across a number of scenarios and policy alternatives. Whereas other studies predict that consumers will wait to buy deferred annuities until very late in life, here we show that investors will optimally purchase measurable amounts of GMWBs well before retirement because of their flexibility and the fact that they offer access to the stock market. Our results indicate that policyholders will exercise this flexibility by taking withdrawals to adjust their portfolios and consumption streams along the way. Nevertheless, at retirement, they also convert much of their accumulated amounts into additional annuities. Moreover, heterogeneity analysis suggests that differences in individuals’ cash out and annuitization patterns result from variations in realized cumulative equity market return and labor income trajectories.

Policymakers are likely to find our analysis useful, as several have expressed interest in products that integrate lifetime income protection into defined contribution pensions. Regulators may also benefit from a clearer assessment of risks associated with individual retirement accounts, along with a possible role for regulation to protect individuals from the downside risk of fluctuating capital markets and the risk of running out of money.
References


